The new riddle of induction

tea talk at Wigner Research Centre for Physics

the “old” riddle

DEDUCTION

general

↓

specific

DEDUCTION

general

↓

specific

all swan are white

A is a swan

DEDUCTION

general

specific

all swan are white

A is a swan

A is white

INDUCTION

specific

general

A is white

A is a swan

---

all swan are white

INDUCTION

specific

↓

general

A is white

A is a swan

B is white

B is a swan

---

all swan are white

the sun came up today

the sun came up yesterday

the sun came up the day before yesterday the sun came up today

the sun came up yesterday

the sun came up the day before yesterday

---

the sun will come up tomorrow

 is a tufa

is a tufa

is a tufa

is a tufa

is a tufa

is a tufa

is a tufa

is a tufa

is a tufa

is a tufa

the “old” riddle:

how are these kinds of inferences justified?

the “old” riddle:

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uniformity of nature

the “old” riddle:

how are these kinds of inferences justified?

uniformity of nature

how is uniformity of nature justified?

the new riddle

that is, are emeralds green or grue?

the new riddle

that is, are emeralds green or grue?

Nelson Goodman 1983
Fact, Fiction and Forecast

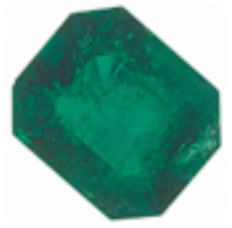
 a green emerald

a green emerald

a green emerald

a green emerald

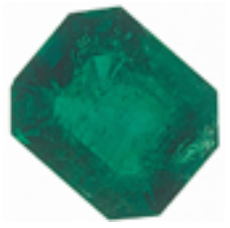
a green emerald



a green emerald

 a green emerald

 a green emerald

 a green emerald

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emeralds are green (evidence supports theory)

DICTIONARY

bleen

if observed before t, blue; else, green

grue

if observed before t, green; else, blue

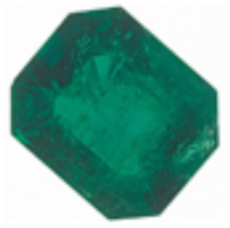
a grue emerald

a grue emerald

a grue emerald

a grue emerald

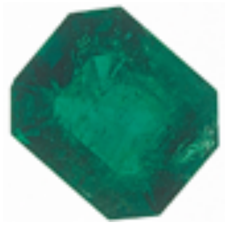
a grue emerald



a grue emerald


a grue emerald


a grue emerald


a grue emerald

---

emeralds are grue (evidence supports theory)

DICTIONARY

green

green

grue

if observed before t, green; else, blue

DICTIONARY

green

green

elaborate definition, use Occam's razor

grue

if observed before t, green; else, blue

DICTIONARY	MARTIAN'S DICTIONARY
green green	**grue** grue
grue if observed before t, green; else, blue	**green** if observed before t, grue; else, bleen

DICTIONARY	MARTIAN'S DICTIONARY
green green	**grue** grue
grue if observed before t, green; else, blue	**green** if observed before t, grue; else, bleen

elaborate definition

- “The US forces were always commanded by George Washington, hence they will be commanded by him in the future”

- “The US forces were always commanded by George Washington, hence they will be commanded by him in the future”
- “The US forces were always commanded by the US president, hence they will be commanded by him in the future”

- “The US forces were always commanded by George Washington, hence they will be commanded by him in the future”
- “The US forces were always commanded by the US president, hence they will be commanded by him in the future”

- “Mary Ball Washington was always the mother of George Washington, hence she will be his mother in the future”

- “The US forces were always commanded by George Washington, hence they will be commanded by him in the future”
- “The US forces were always commanded by the US president, hence they will be commanded by him in the future”

- “Mary Ball Washington was always the mother of George Washington, hence she will be his mother in the future”
- “Mary Ball Washington was always the mother of the US president, hence she will be his mother in the future”

- inductive inference is relative to the language it is formulated in

BAYESIAN MODEL SELECTION

BAYESIAN MODEL SELECTION

H1: emeralds are green

H2: emeralds are grue

BAYESIAN MODEL SELECTION

posterior
over
models ←

$$P(\mathcal{H}_i \,|\, D) \propto P(D \,|\, \mathcal{H}_i) P(\mathcal{H}_i)$$

BAYESIAN MODEL SELECTION

posterior over models

$$P(\mathcal{H}_i \,|\, D) \propto P(D \,|\, \mathcal{H}_i) P(\mathcal{H}_i)$$

evidence compatible with both grue and green

BAYESIAN MODEL SELECTION

$$P(\mathcal{H}_i \,|\, D) \propto P(D \,|\, \mathcal{H}_i) P(\mathcal{H}_i)$$

posterior
over
models

evidence
compatible
with both
grue and
green

model prior
decides

MINIMUM DESCRIPTION LENGTH

the best model is the one that leads to the best compression of the observed data

MINIMUM DESCRIPTION LENGTH

$$L(D, \mathcal{H}) = L(\mathcal{H}) + L(D \,|\, \mathcal{H})$$

MINIMUM DESCRIPTION LENGTH

description length ← $$L(D, \mathcal{H}) = L(\mathcal{H}) + L(D \,|\, \mathcal{H})$$

MINIMUM DESCRIPTION LENGTH

$$L(D, \mathcal{H}) = L(\mathcal{H}) + L(D \,|\, \mathcal{H})$$

description length

length of model specification

MINIMUM DESCRIPTION LENGTH

$$L(D, \mathcal{H}) = L(\mathcal{H}) + L(D \,|\, \mathcal{H})$$

description length

length of model specification

length of data specification given model

KOLMOGOROV COMPLEXITY

10101010101010101010101010101010101010101010101010

print "01" * 25

KOLMOGOROV COMPLEXITY

10101010101010101010101010101010101010101010101010

$$\text{description}_{\text{PYTHON}} = (\text{program}_{\text{PYTHON}}, \text{ input})$$

$$K_{\text{PYTHON}} = 13$$

depends on language

$$K = L(\text{description}) = L(\text{input}) + \text{const}$$

$$L(D, \mathcal{H}) = L(\mathcal{H}) + L(D \,|\, \mathcal{H})$$

Minimum Description Length

$$\begin{aligned} L(D, \mathcal{H}) &= -\log P(\mathcal{H}) - \log \left(P(D \,|\, \mathcal{H}) \delta D\right) \\ &= -\log P(\mathcal{H} \,|\, D) + \text{const.} \end{aligned}$$

Bayesian inference

$$L(D,\mathcal{H}) = L(\mathcal{H}) + L(D\,|\,\mathcal{H})$$

Minimum Description Length

Kraft inequality

$$P(\mathbf{x}) = 2^{-L(\mathbf{x})}, \quad L(\mathbf{x}) = -\log_2 P(\mathbf{x})$$

$$\begin{aligned} L(D,\mathcal{H}) &= -\log P(\mathcal{H}) - \log\left(P(D\,|\,\mathcal{H})\delta D\right) \\ &= -\log P(\mathcal{H}\,|\,D) + \text{const.} \end{aligned}$$

Bayesian inference

$$L(D, \mathcal{H}) = \boxed{L(\mathcal{H})} + L(D \,|\, \mathcal{H})$$

Minimum Description Length

model language

Kraft inequality

$$P(\mathbf{x}) = 2^{-L(\mathbf{x})}, \quad L(\mathbf{x}) = -\log_2 P(\mathbf{x})$$

$$\begin{aligned} L(D, \mathcal{H}) &= -\log \boxed{P(\mathcal{H})} - \log \left(P(D \,|\, \mathcal{H}) \delta D \right) \\ &= -\log P(\mathcal{H} \,|\, D) + \text{const.} \end{aligned}$$

Bayesian inference

TURING MACHINES

“the subject must pick a (*universal*) Turing machine whose operations describe the basic operations believed to represent "simplicity" by the subject.

However, one could always choose a Turing machine with a simple operation that happened to construct one's entire theory and would hence score highly under the razor.”

- Goodman’s problem - inductive inference is relative to the language it is formulated in

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- this problem appears in the formal approaches as well

- Goodman's problem - inductive inference is relative to the language it is formulated in

- this problem appears in the formal approaches as well

- how should we choose the language?

SOURCES

http://plato.stanford.edu/entries/induction-problem/#GruParNewRidInd

David McKay - Information Theory, Inference, and Learning Algorithms

http://jeremykun.com/2012/04/21/kolmogorov-complexity-a-primer/

http://en.wikipedia.org/wiki/Occam's_razor

- What is the difference between those generalizations that are supported by their instances and those that are not?
- Which generalizations support counterfactual conditionals?
- How are lawlike generalizations to be distinguished from accidental generalizations?