A simple classical explanation for question order effects

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Abstract

One of the main pieces of experimental evidence supporting the quantum cognition hypothesis is given in [Wang2014]. Here I wish to show that the effects considered there also arise in classical probabilistic models with minimal assumptions.

Probabilistic model of survey answers

We denote the distribution of answers to a question in a general context by $P(A|\ general\ context)$. If some related question is asked before this, then the distribution of answers to the second question (denoted by $P(A|\ modified\ context)$) is likely to be different. The magnitude of this difference, denoted by Δ_1, is a function of how related the questions are. If the questions have nothing to do with each other, then $\Delta \approx 0$ and there is no order effect. For pairs of questions that don't completely change each other's interpretations, we can assume that Δ is small. We can summarize the above in the following equations for a given pair of questions A and B:

$$P(A = yes \mid general\ context) = p_a$$

$$P(A = yes \mid modified\ context) = p_a + \Delta_a$$

$$P(B = yes \mid general\ context) = p_b$$

$$P(B = yes \mid modified\ context) = p_b + \Delta_b$$

To avoid having to deal with what the questions are exactly, we generate these probabilities randomly. However, we want to make sure that we generate probabilities that are representative of the kinds of questions considered here. In general we expect that while for one question the answers might be biased towards yes, for others it will be the opposite. However, there is no reason to think that most questions are going to be biased towards either yes or no, so we assume that the distribution of p_i-s is symmetric (*assumption 1*). Since probabilities have to be between 0 and 1, we need a symmetric distribution over $[0, 1]$. A simple parametric family that satisfies these assumptions is the beta distribution with identical parameters.

$$p_i \sim \mathrm{Beta}(\beta, \beta)$$

Since the questions are from surveys, we expect that they are chosen such that the answers are not completely obvious for everyone: for most questions, the proportion of people that answer yes is closer to 1/2 than zero or one (*assumption 2*). This is true for all $\beta > 1$, and the extent of this preference increases with β. Differences in the distribution of answers due to the modified context (Δ_i) can also go either way (*assumption* 3) and they are typically small (*assumption* 4). This can be fulfilled by taking a distribution of differences that is symmetric and has small variance ϵ, such as

$$\Delta_i \sim Normal(0, \epsilon).$$

Results

It can be shown that the QQ equality is a straightforward consequence of these assumptions. If we take the above distributions for p_i and Δ_i, then the correlation between the first and the second context effect is

$$Corr[AyBy - ByAy,\ AnBn - BnAn] = -\frac{\beta_1}{\beta_1 + 1},$$

which goes to -1 for $\beta \to \infty$. For the derivation of this result, see the appendix. The order effect can also be reproduced in this model. The fact that the q/order ratio is strictly lower than one is a simple consequence of the triangle inequality:

$$|\Delta_1 - \Delta_2 + 2\Delta_2 p_1 - 2\Delta_1 p_2| \leq |\Delta_1 - \Delta_2 + \Delta_2 p_1 - \Delta_1 p_2| + |\Delta_2 p_1 - \Delta_1 p_2|$$

We can see if our assumptions generate similar data to those that were recorded in the surveys by sampling questions from our distributions and producing the plots used in [Wang2014]. Figure 1 and 2 show the QQ plot and the order effect respectively. An exploration of the variance in these plots as a function of the free parameters (β and ϵ) of the model are also provided in the appendix.

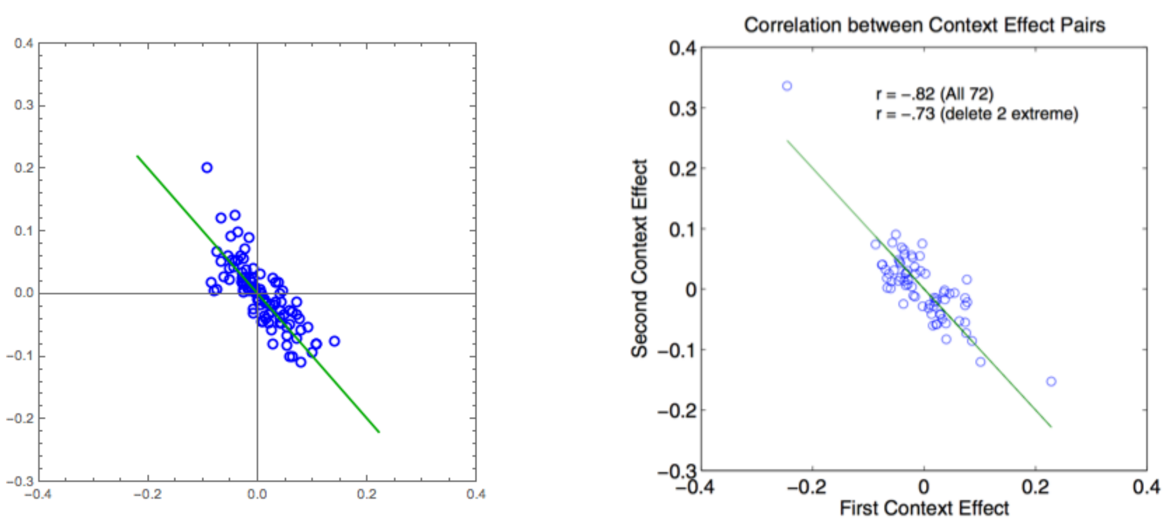
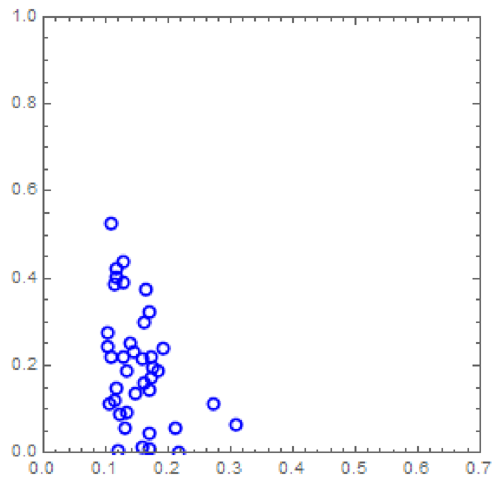


Figure 1: *Left*, sampled QQ plot from model with $\beta = 5$ and $\epsilon = 0.07$. *Right*, original QQ plot reproduced from [Wang2014].



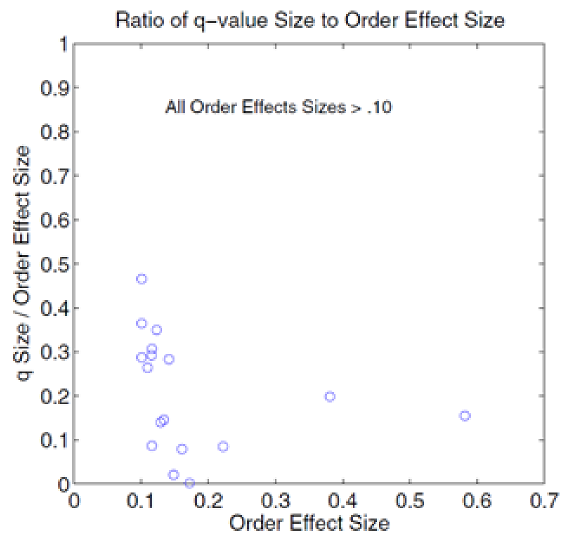


Figure 2: *Left*, order effect for sampled points with same parameters as in figure 1. *Right*, original order effect plot reproduced from [Wang2014].

An example:

- $P(Clinton|Everyone) = p_1$
- $P(Clinton|Politicians) = P(Clinton|Everyone) + \Delta_1$
- $P(Gore|Everyone) = p_2$
- $P(Gore|Politicians) = P(Gore|Everyone) + \Delta_2$

References

[Lanyon2010] Lanyon, Benjamin P., et al. "Towards quantum chemistry on a quantum computer." Nature Chemistry 2.2 (2010): 106-111.

[Wang2014] Wang, Zheng, et al. "Context effects produced by question orders reveal quantum nature of human judgments." Proceedings of the National Academy of Sciences 111.26 (2014): 9431-9436.

Appendix

$$Corr[(\Delta_2 p_1 - \Delta_1 p_2), (\Delta_1 - \Delta_2 + \Delta_2 p_1 - \Delta_1 p_2)]$$

$$E[\Delta_1\Delta_2 p_1 - \Delta_2^2 p_1 + \Delta_2^2 p_1^2 - \Delta_1^2 p_2 + \Delta_1\Delta_2 p_2 - 2\Delta_1\Delta_2 p_1 p_2 + \Delta_1^2 p_2^2] = -\frac{\beta\epsilon^2}{2\beta+1}$$

$$\sqrt{E[(\Delta_2 p_1 - \Delta_1 p_2)^2]} = \sqrt{E[(\Delta_1 - \Delta_2 + \Delta_2 p_1 - \Delta_1 p_2)^2]} = \sqrt{\frac{(1+\beta)\epsilon^2}{1+2\beta}}$$

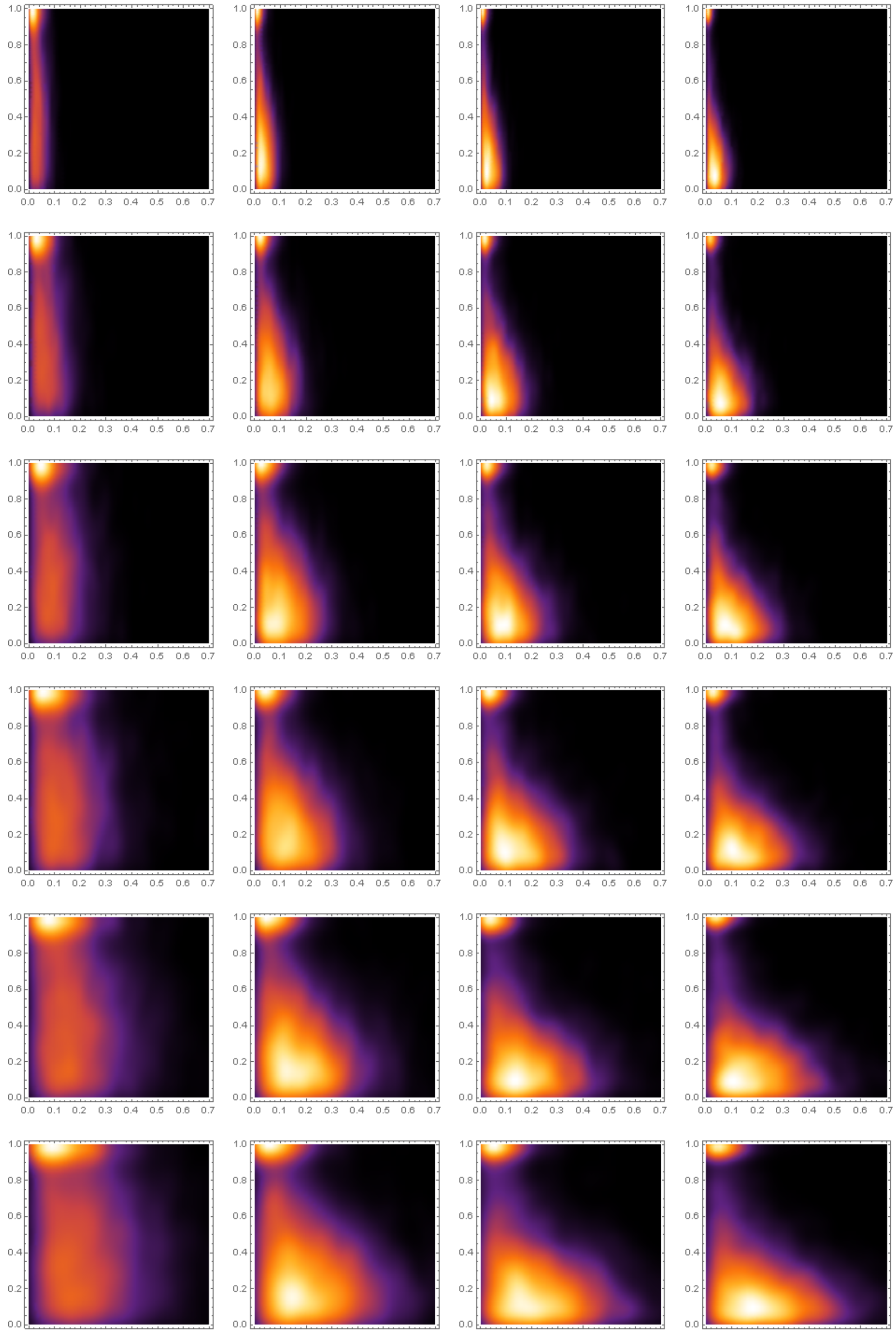


Figure 3: β horizontally from left to right: (1,3.5,6,8.5), ϵ from 0.03 to 0.18 by steps of 0.03

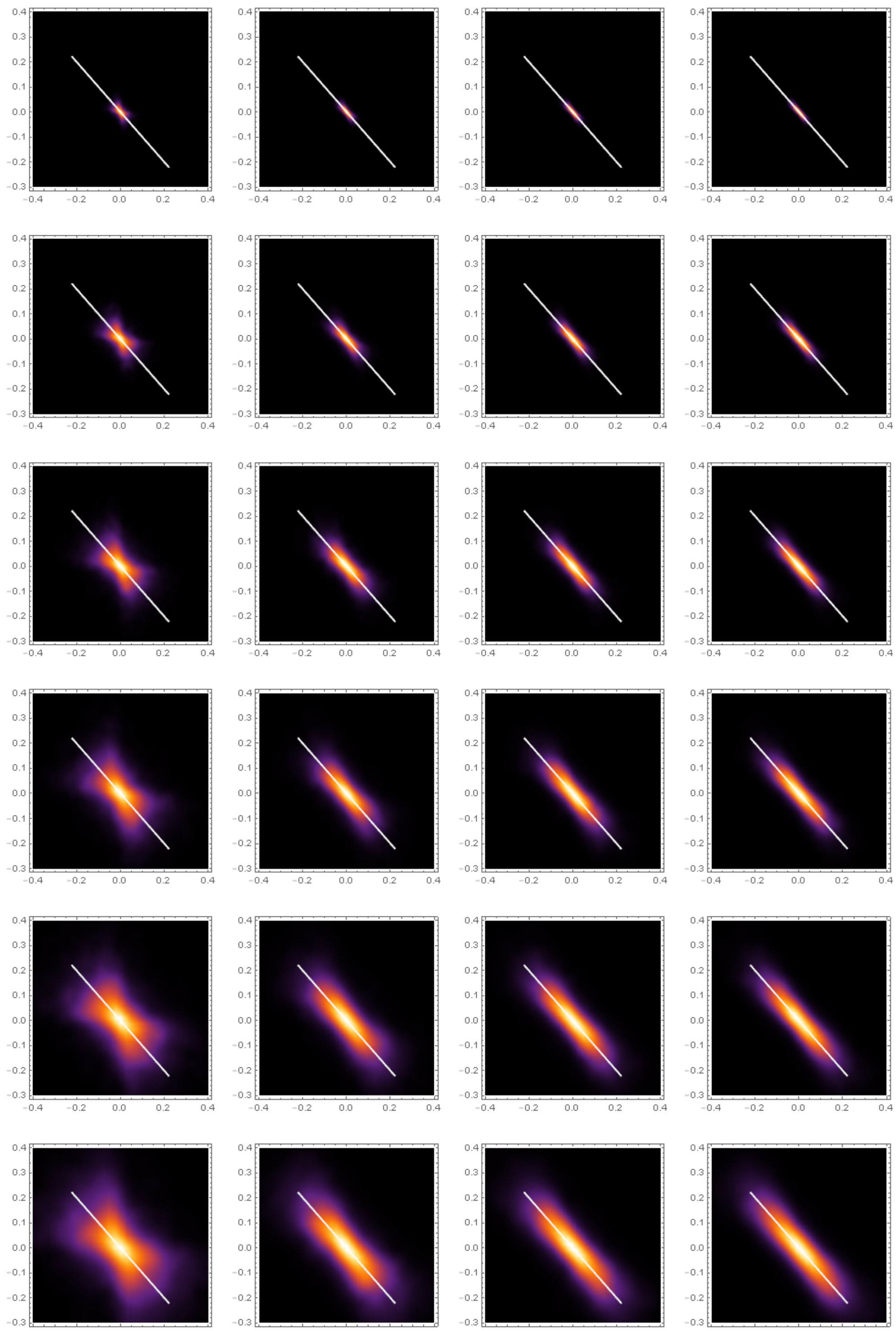


Figure 4: β horizontally from left to right: (1,3.5,6,8.5), ϵ from 0.03 to 0.18 by steps of 0.03