Knowledge representation

David Nagy
davidnagy@elte.hu

Statistical learning in the nervous system

golab.wigner.mta.hu/teaching

previously

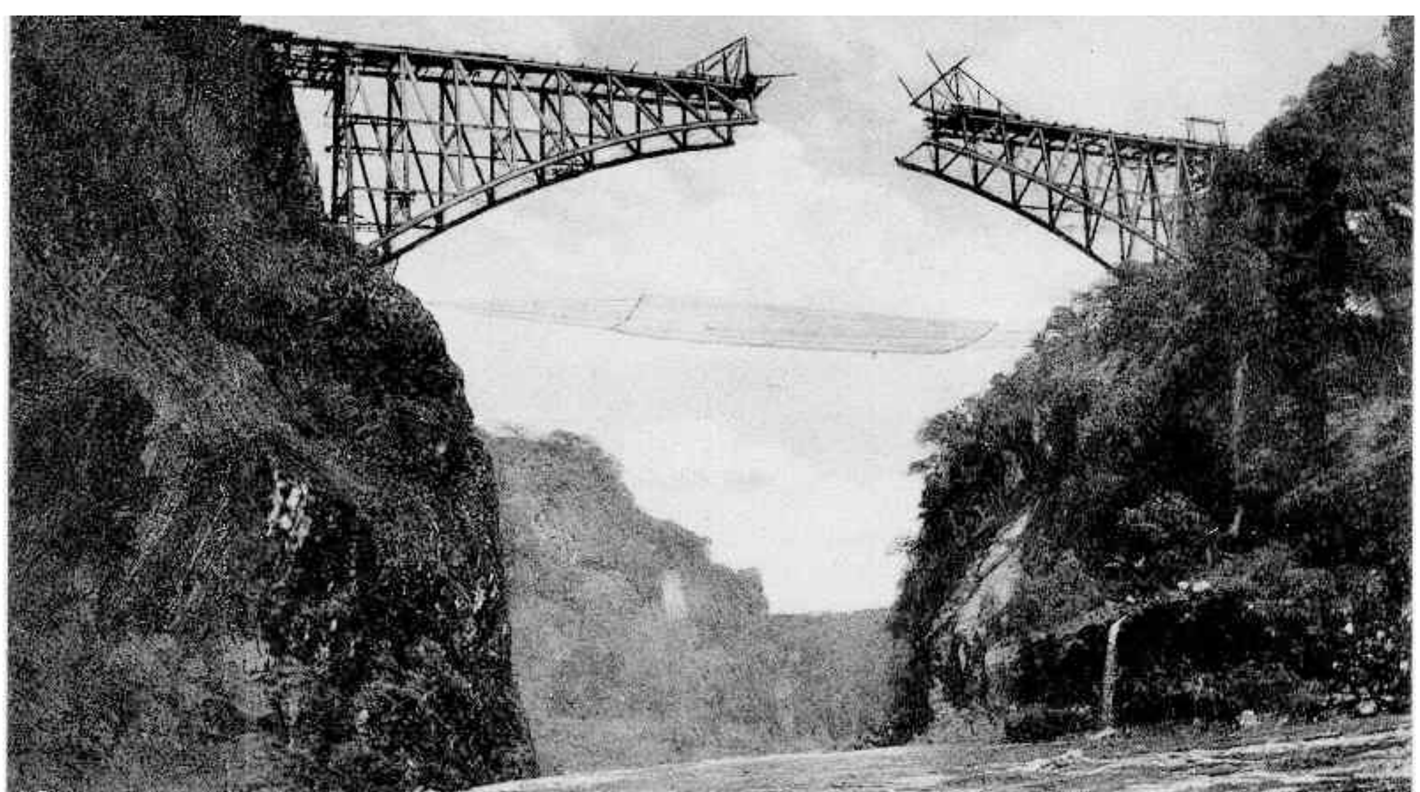
- **Computation** - determining the challenge:
  through the goals of the biological agent phrasing a mathematical model that is capable of addressing the challenge

- **Algorithm** - solving the challenge can be achieved in many different ways that can be weighed based on different factors

- **Implementation** - physical realization of the algorithm, in the case of neuroscience, based on neurons, spikes, etc

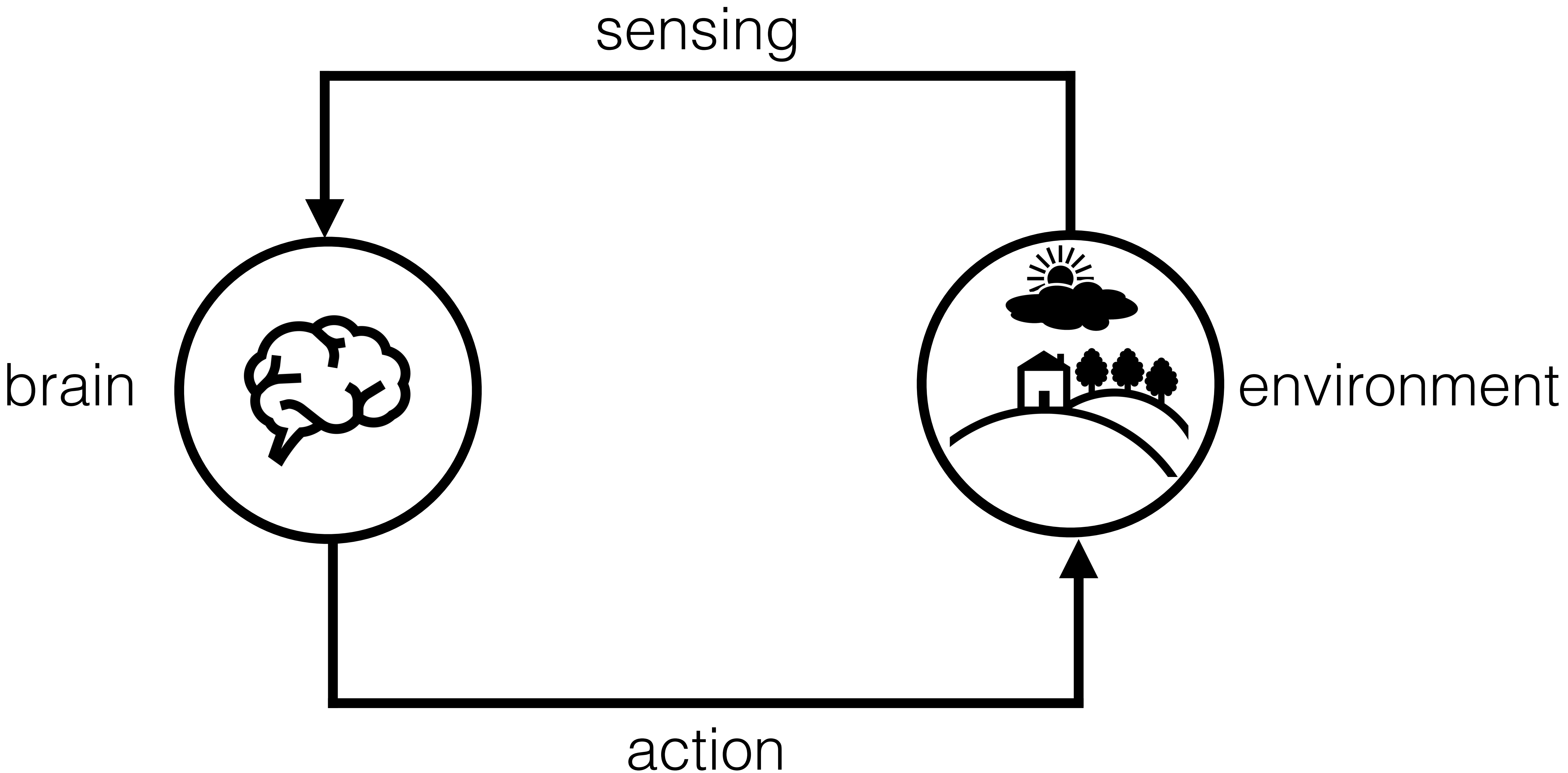
bottom-up

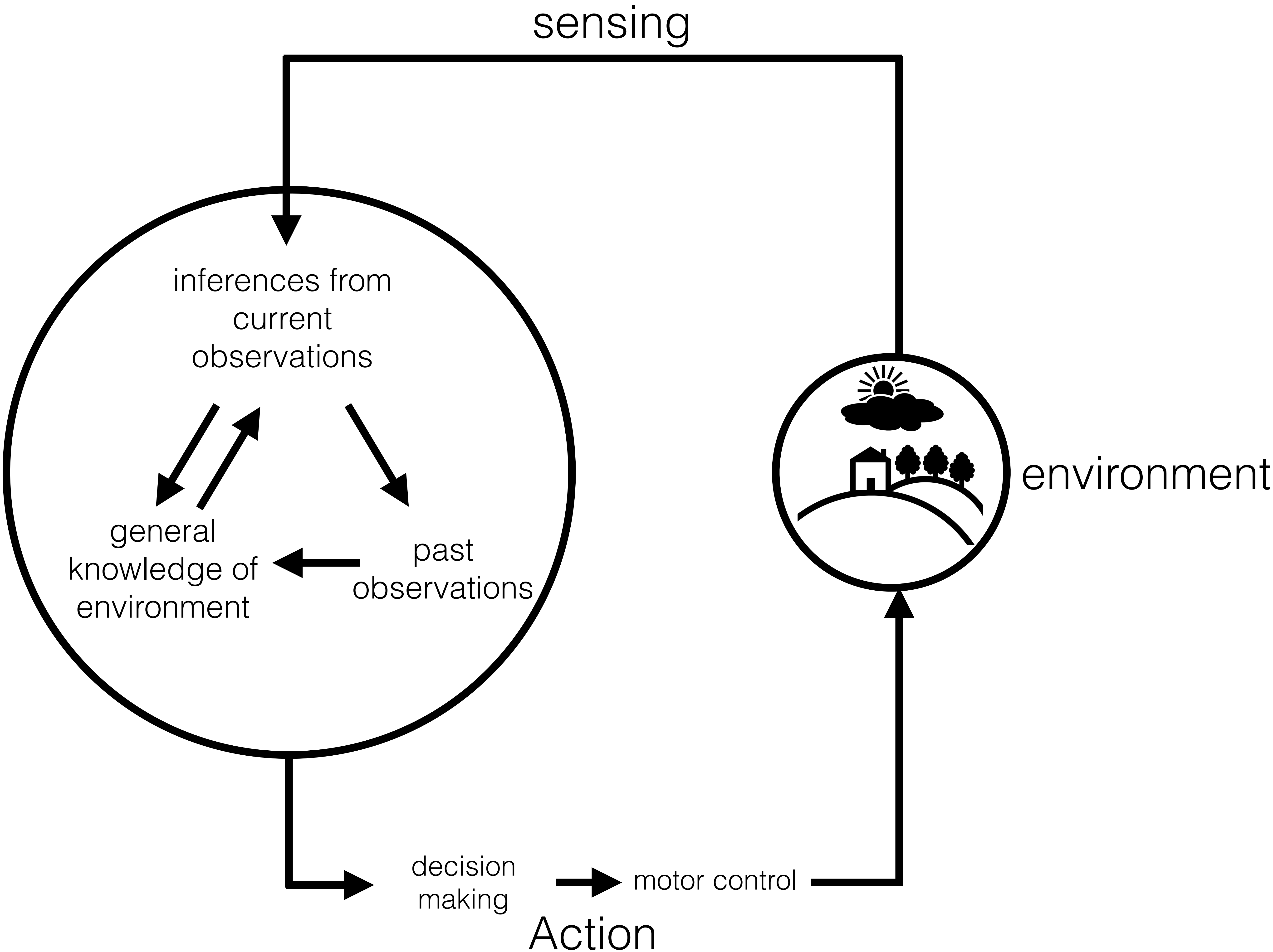
top-down

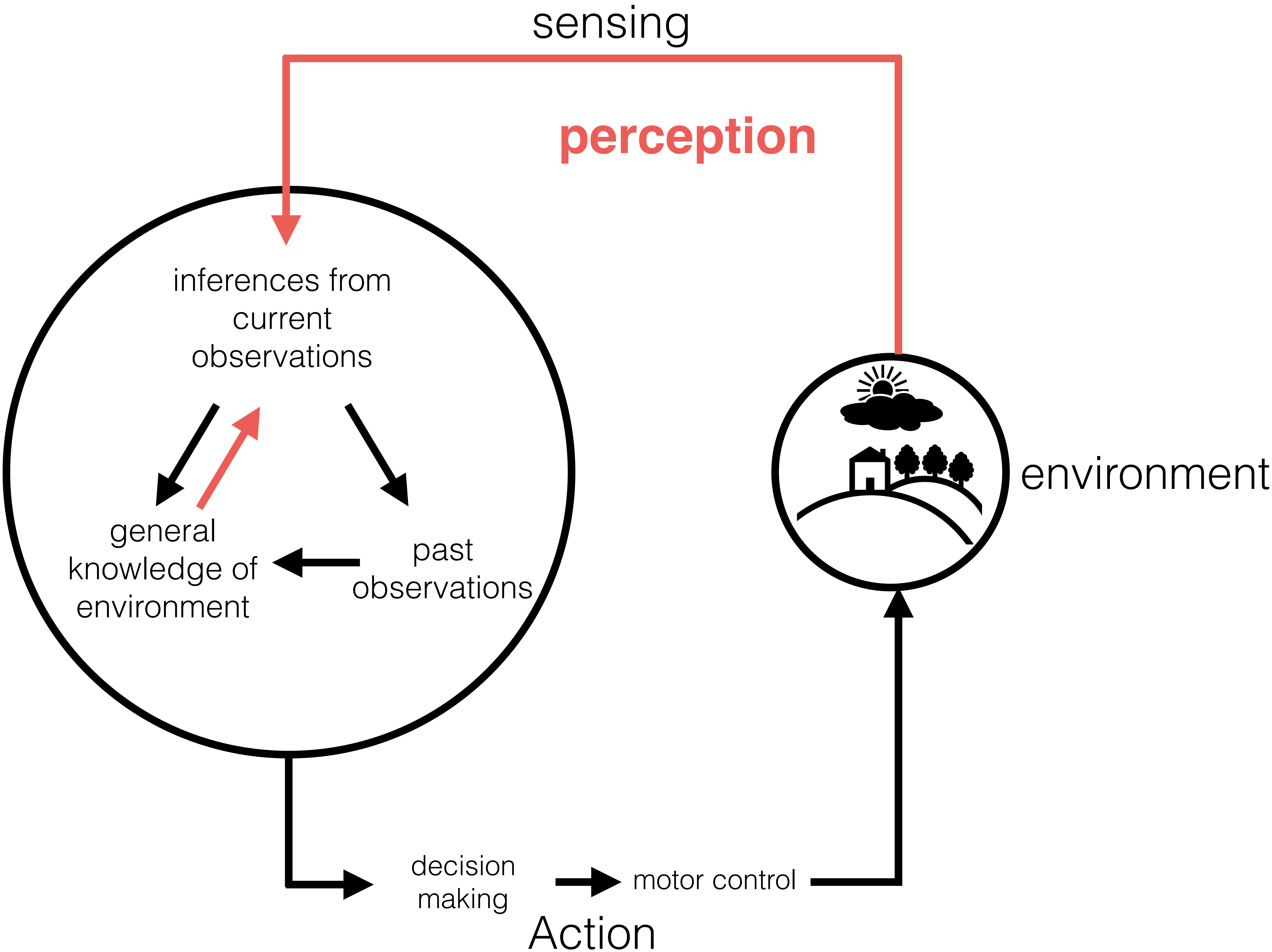


David Marr, 1976

previously




sensing
inferences from
current
observations
general
knowledge of
environment
past
observations
environment
decision
making
motor control
Action


sensing
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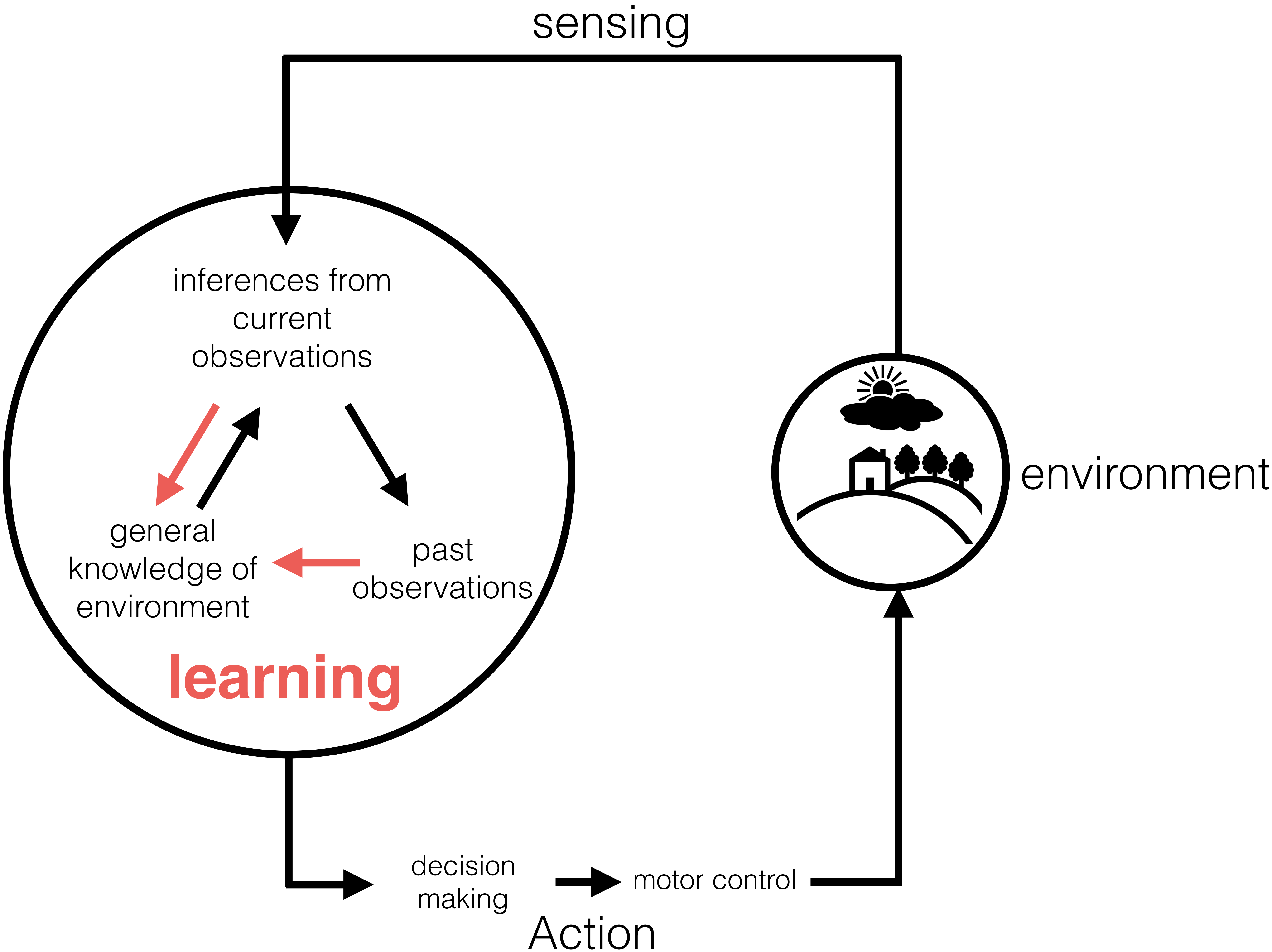

sensing
inferences from current observations
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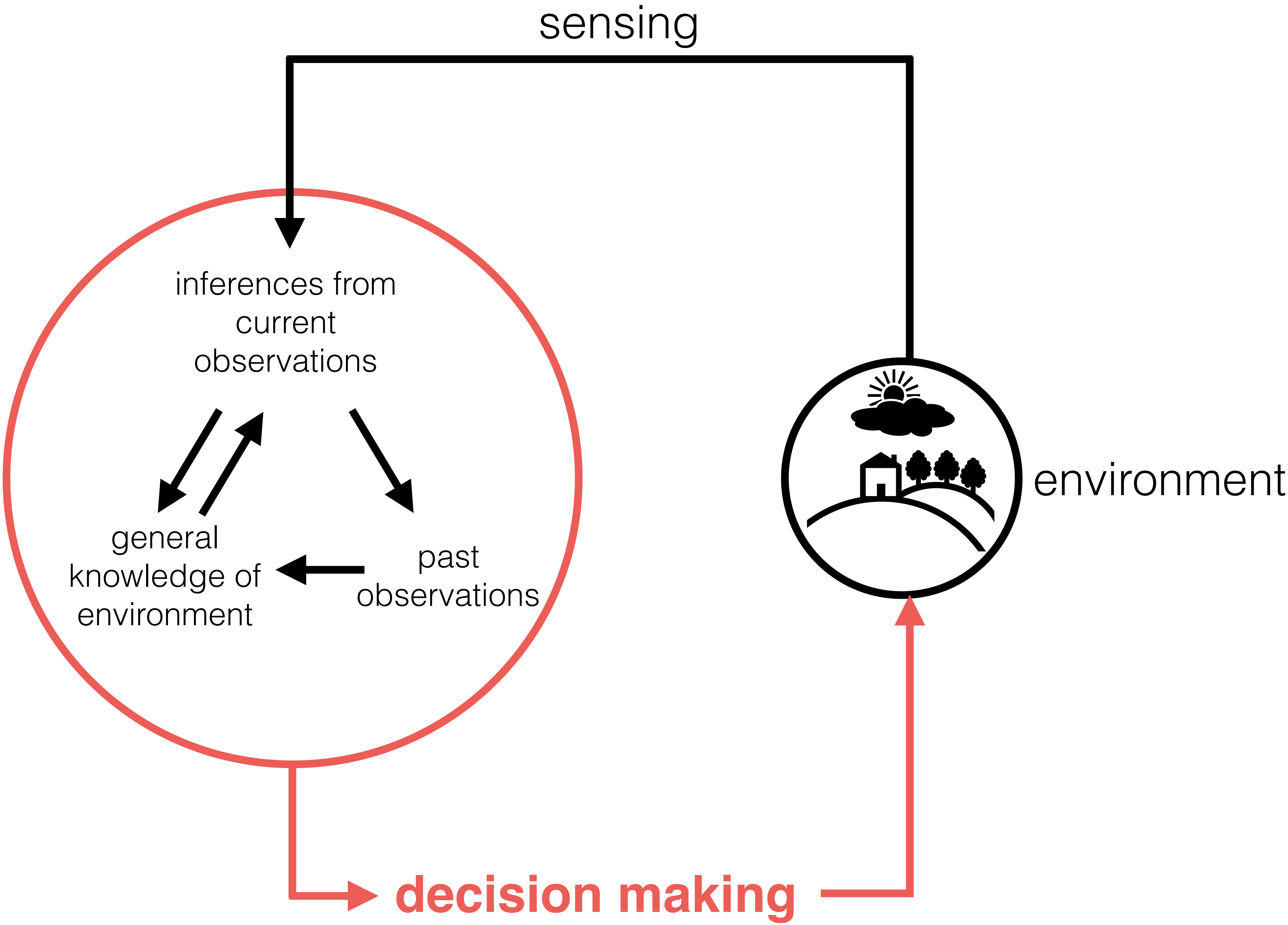
sensing
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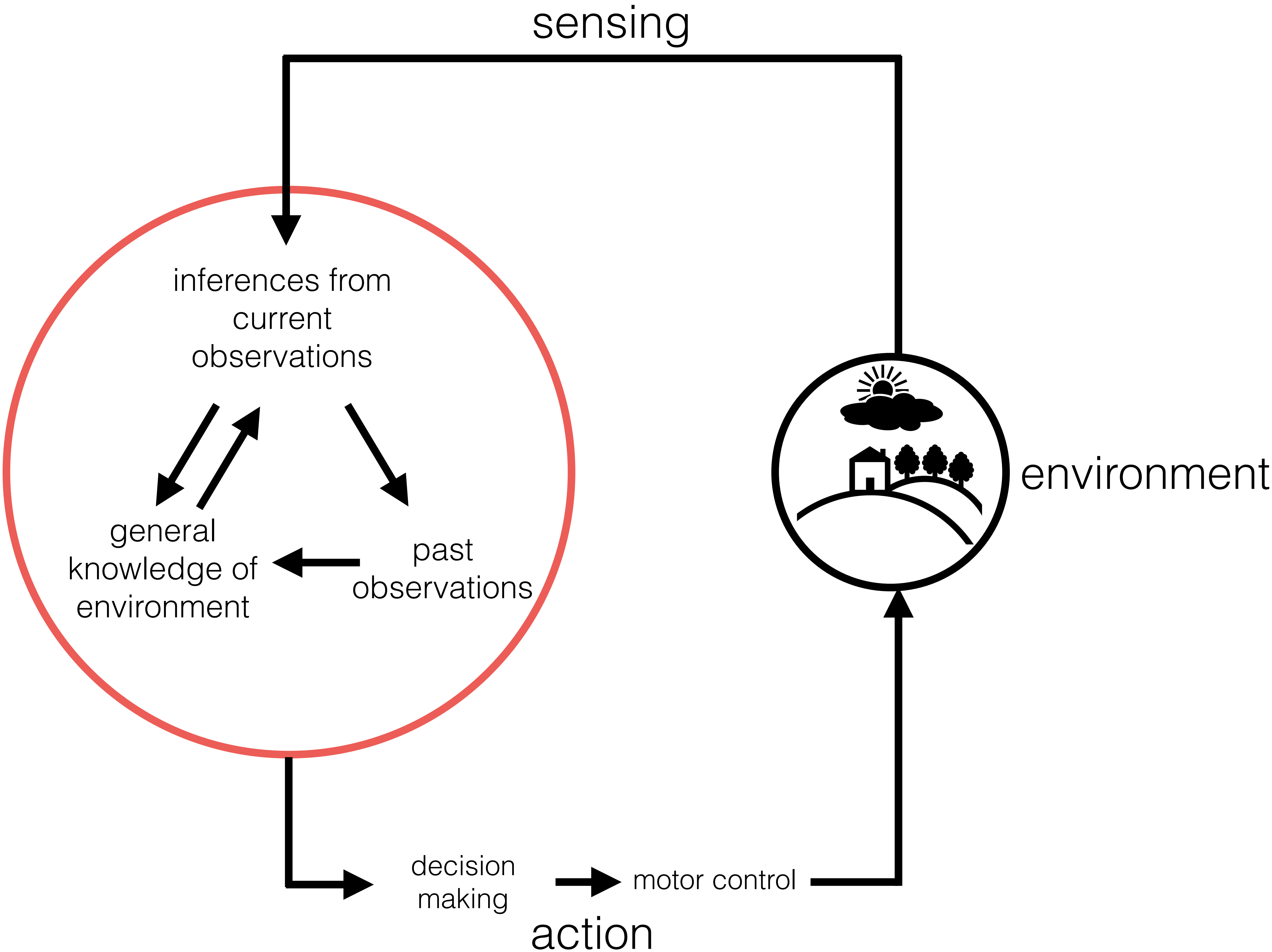
sensing
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action

goals for today

- what is knowledge?
- what are internal models?
- what are the limits of representing knowledge with neurons?
- why are we going to use probability theory as a mathematical framework in the rest of the course?

goals for today

- **what is knowledge?**
- what are internal models?
- what are the limits of representing knowledge with neurons?
- why are we going to use probability theory as a mathematical framework in the rest of the course?

what is knowledge?

- how do we test knowledge?

Kötelező évszámok

Őskor, ókori Kelet:

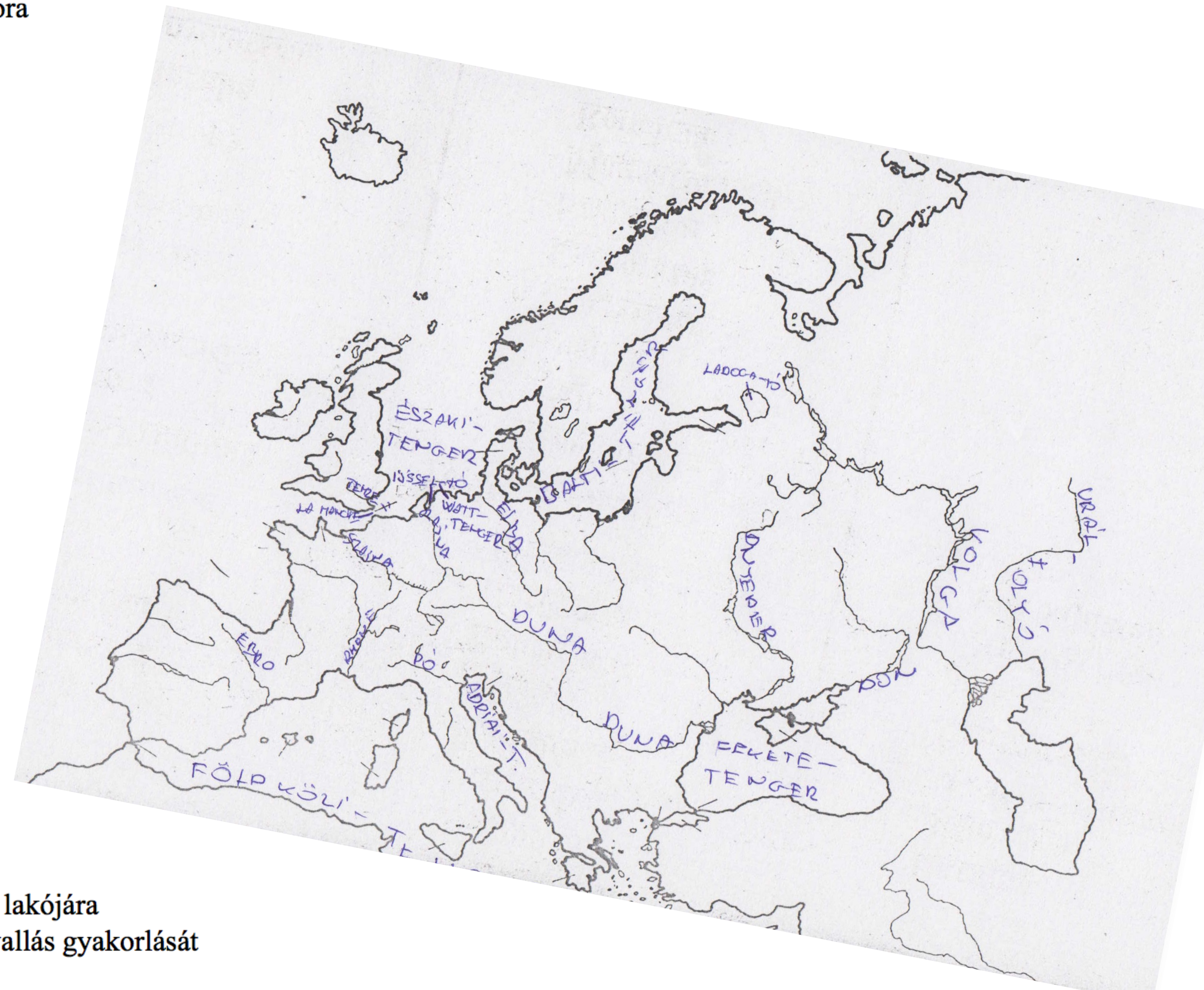
100ezer (30ezer) éve – a Homo Sapiens megjelenése
Kr. e. 3000 körül – Alsó- és Felső Egyiptom egyesülésével létrejön az ókori egyiptomi állam
Kr. e. XVIII. sz. – Hammurapi uralkodása, az Óbabiloni Birodalom fénykora
Kr. e. X. sz. – a zsidó állam fénykora Dávid és Salamon idején
Kr. e. 525 – péliuszioni csata, Egyiptom bukása (perzsa uralom alá kerül)

Az ókori Görögország:

Kr. e. XIII. sz. – a dórok támadása, a mükénéi kultúra bukása
Kr. e. 621 – Drakón írásba foglalja a törvényeket Athénban
Kr. e. 508 – Kleiszthenész reformjai Athénban, a demokrácia kialakulása
Kr. e. 480 – szalamiszi csata
Kr. e. 480 – thermopülei csata
Kr. e. 776 – az első olimpiai játékok
Kr. e. 431-404 – peloponnészoszi háború
Kr. e. 336-323 – Nagy Sándor uralkodása

Az ókori Róma:

Kr. e. 753 – Róma alapítása
Kr. e. 510 – a királyság bukása, a köztársaság kezdete
Kr. e. 367 – Sextius-Licinius-féle törvények (földtörvény)
Kr. e. 287 – megszűnik a népgyűlés határozatainak senatusi jóváhagyása
Kr. e. 264-241 – az 1. pun háború
Kr. e. 218-201 – a 2. pun háború
Kr. e. 168 – a püdnai csata, Macedonia meghódítása
212 - Caracalla kiterjeszti a római polgárjogot a birodalom minden szabad lakójára
313 - Nagy Constantinus a milánói edictumban engedélyezi a keresztény vallás gyakorlását
395 - a Római Birodalom kettészakadása
476 – a Nyugat-Római Birodalom bukása



1, 2, 3, … , 100

1) $\begin{array}{r} 408 \\ -\ 234 \\ \hline \end{array}$ 2) $\begin{array}{r} 842 \\ -\ 219 \\ \hline \end{array}$ 3) $\begin{array}{r} 729 \\ -\ 145 \\ \hline \end{array}$

4) $\begin{array}{r} 675 \\ -\ 232 \\ \hline \end{array}$ 5) $\begin{array}{r} 903 \\ -\ 81 \\ \hline \end{array}$ 6) $\begin{array}{r} 673 \\ -\ 528 \\ \hline \end{array}$

7) $\begin{array}{r} 768 \\ -\ 237 \\ \hline \end{array}$ 8) $\begin{array}{r} 892 \\ -\ 79 \\ \hline \end{array}$ 9) $\begin{array}{r} 758 \\ -\ 573 \\ \hline \end{array}$

10) $\begin{array}{r} 604 \\ -\ 68 \\ \hline \end{array}$ 11) $\begin{array}{r} 786 \\ -\ 471 \\ \hline \end{array}$ 12) $\begin{array}{r} 774 \\ -\ 556 \\ \hline \end{array}$

9. Vízcseppek potyogtak a papírra. Írd be, mik lehettek az elmosódott számok!

$\begin{array}{r} 34678 \\ +\ 97862 \\ \hline 132540 \end{array}$ $\begin{array}{r} 63123 \\ +\ 7097 \\ \hline 70220 \end{array}$ $\begin{array}{r} 55602 \\ +\ 23412 \\ \hline 79014 \end{array}$ $\begin{array}{r} 11111 \\ +\ 6666 \\ \hline 17777 \end{array}$

$\begin{array}{r} 51239 \\ +\ 22222 \\ \hline 73461 \end{array}$ $\begin{array}{r} 8230 \\ +\ 93401 \\ \hline 101631 \end{array}$ $\begin{array}{r} 5555 \\ +\ 9001 \\ \hline 14556 \end{array}$ $\begin{array}{r} 4512 \\ +\ 9012 \\ \hline 13524 \end{array}$

what is knowledge?

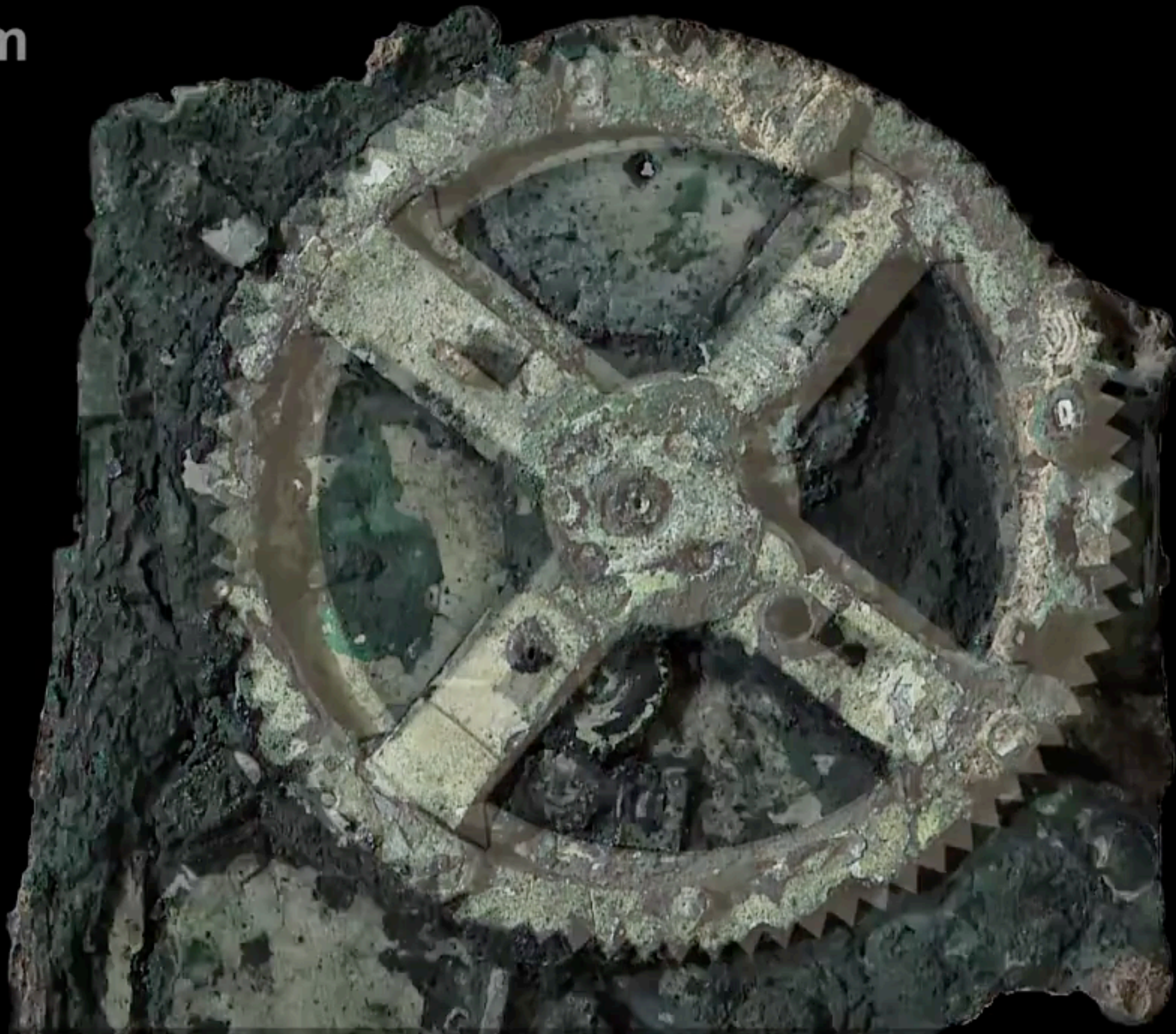
- how do we test knowledge?
    - can solve previously unseen examples (e.g. addition)
    - can use dates and other knowledge for new inferences
    - can answer “what if” or “what is going to happen” questions

what is knowledge?

- But we also call it knowledge if the subject
  - can raw an elephant
  - has a good chance of hitting a ball back with a tennis racket
  - can plan quick trips within bp using public transport
  - can tell you the rules of chess
  - can play chess well

what is knowledge?

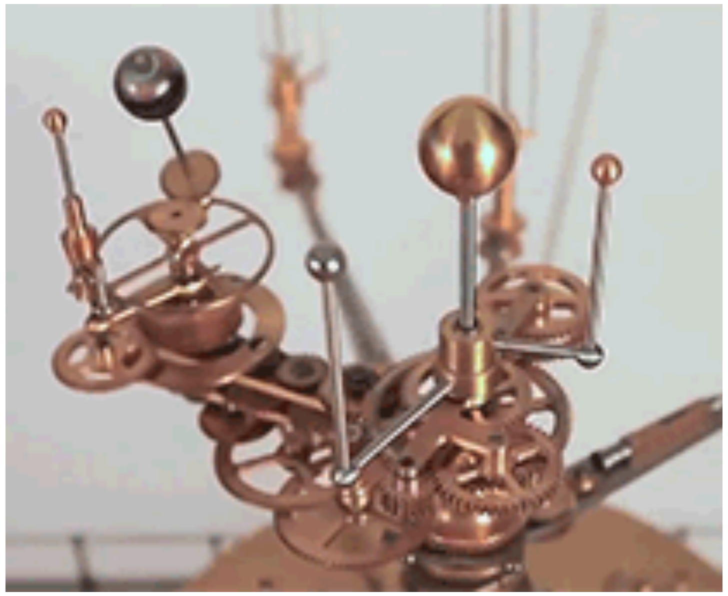
- taken together, we have covered a large proportion of possible tests of knowledge.
- assuming that we think of the brain as a physical system,
- how could we build a physical system that would pass these tests?
  - let us look at an early example where we are interested in the movements of celestial bodies such as the sun and the moon



Antikythera mechanism

- we are interested in the movements of the planets, the sun and the moon
- highlight parts of the real system that are relevant from this point of view
    - e.g. neglect dark matter, atmospheres of planets, inner structure of planets etc.

Antikythera mechanism



- we build a **model**
  - a machine where the parts we have identified with relevant variables in the real system work in approximately the same way
- then, we can operate this machine to answer questions about the real system
  - e.g. to predict eclipses
- what are the limits to constructing such models?

Can a machine simulate another machine?

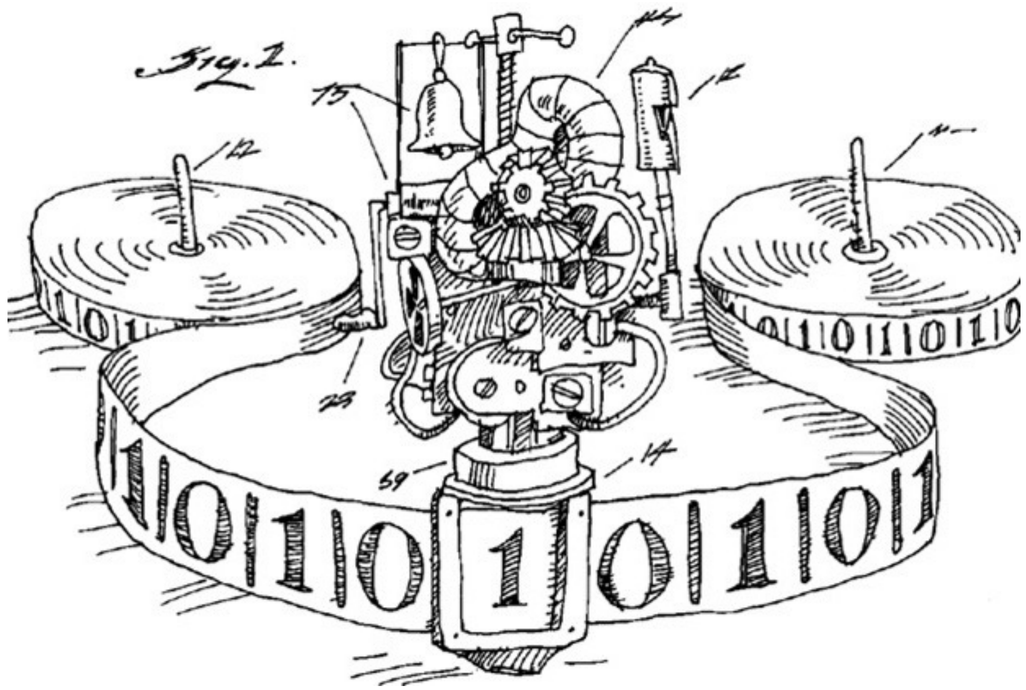

1500 Lego Technic parts
Eclipse Cycle
Calendar Dial

Can a machine compute something that no other machine can simulate?



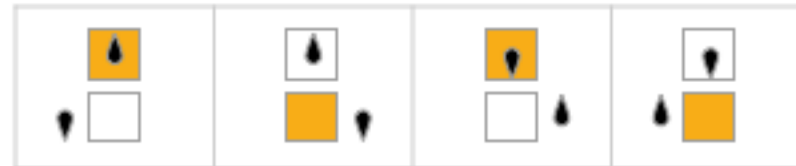
COMPUTING DIVISION
COMPUTING SECTION

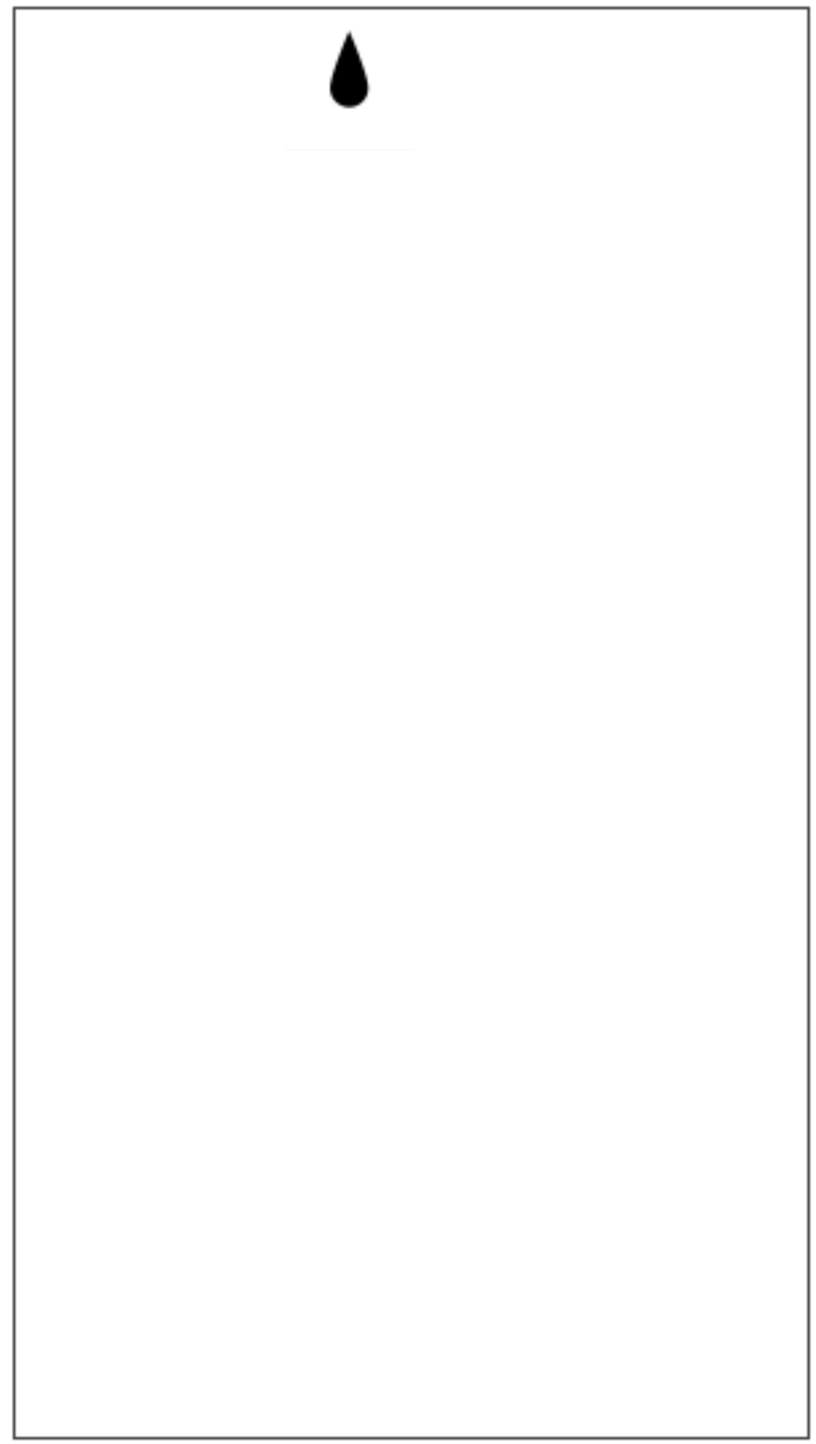
Turing-machine

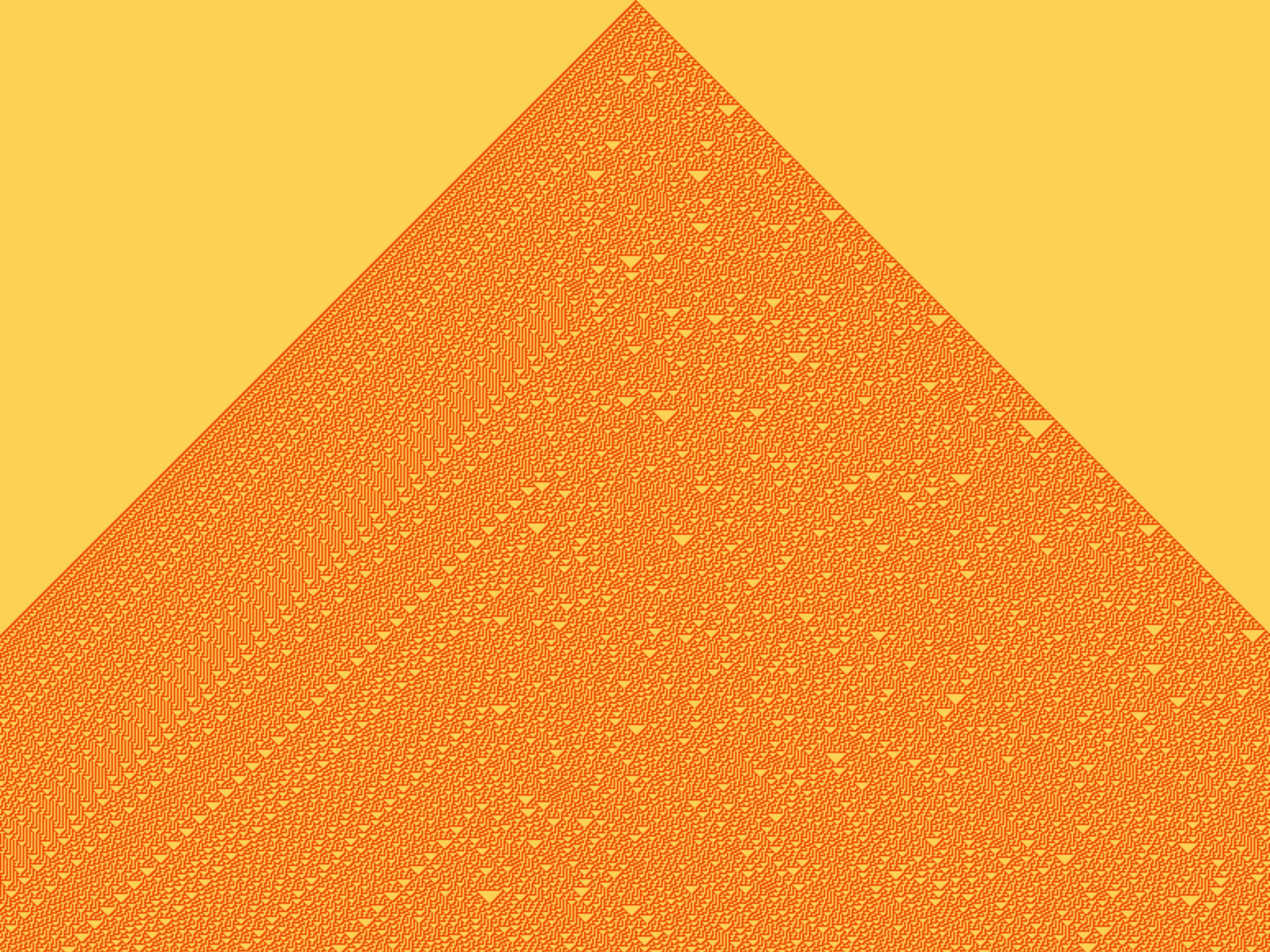


- an infinite tape with cells
- on which it can
    - write symbols
    - erase symbols
    - read symbols
    - move between cells
    - end the computation
- which of these it performs at any time depends on two things:
    - what symbol was read
    - what state the machine is in
- where the appropriate action for each symbol - state pair is given in advance in a table

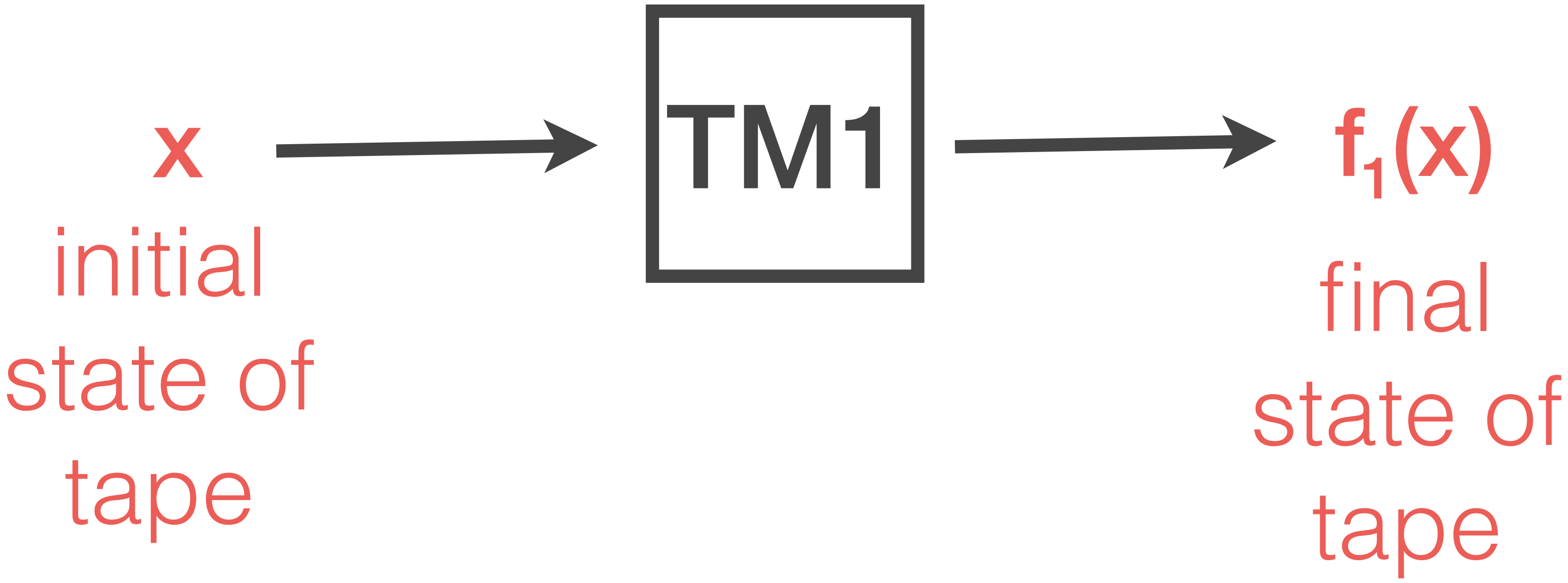
Tape symbol	Current state A			Current state B			Current state C		
	Write symbol	Move tape	Next state	Write symbol	Move tape	Next state	Write symbol	Move tape	Next state
0	1	R	**B**	1	L	**A**	1	L	**B**
1	1	L	**C**	1	R	**B**	1	R	**HALT**



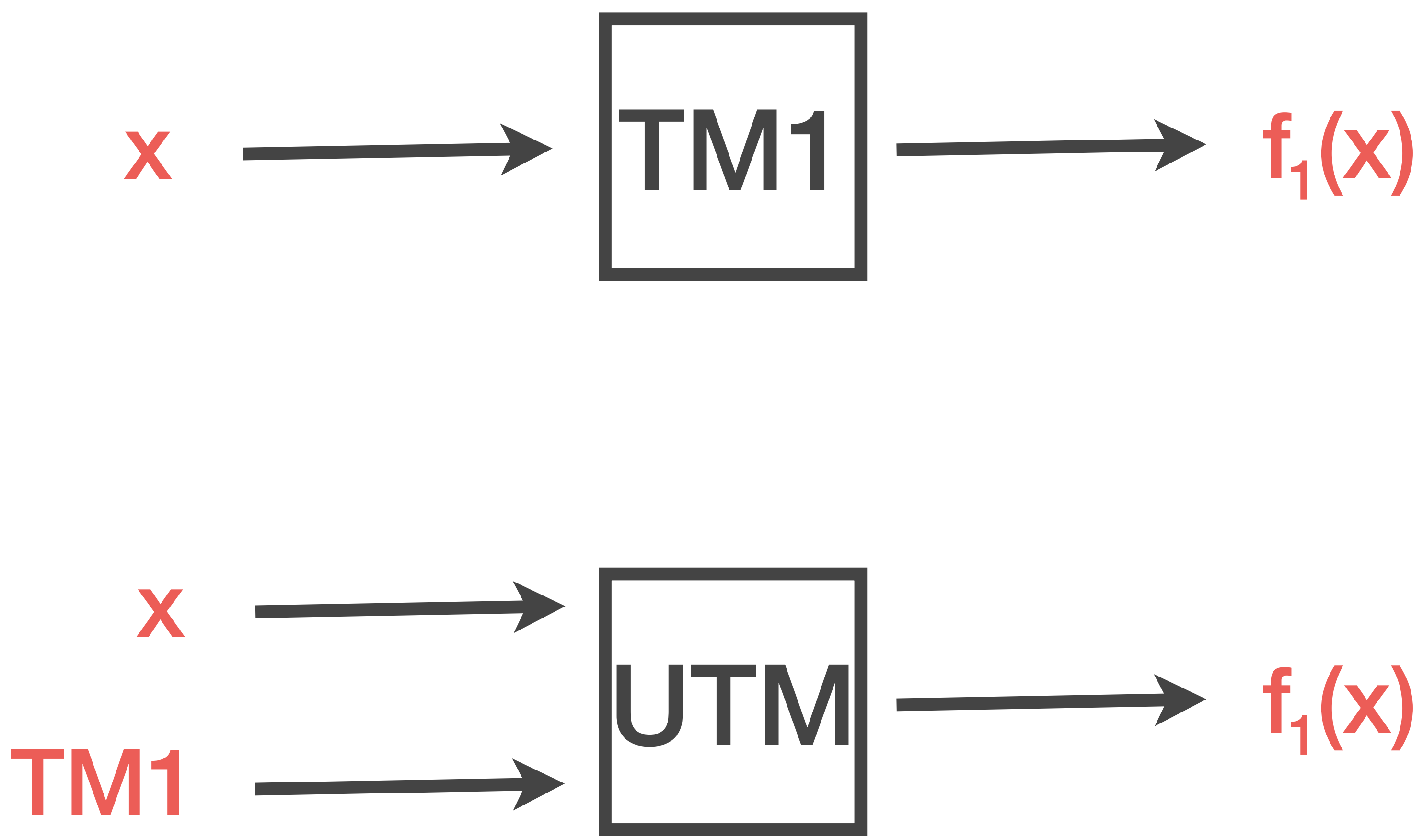


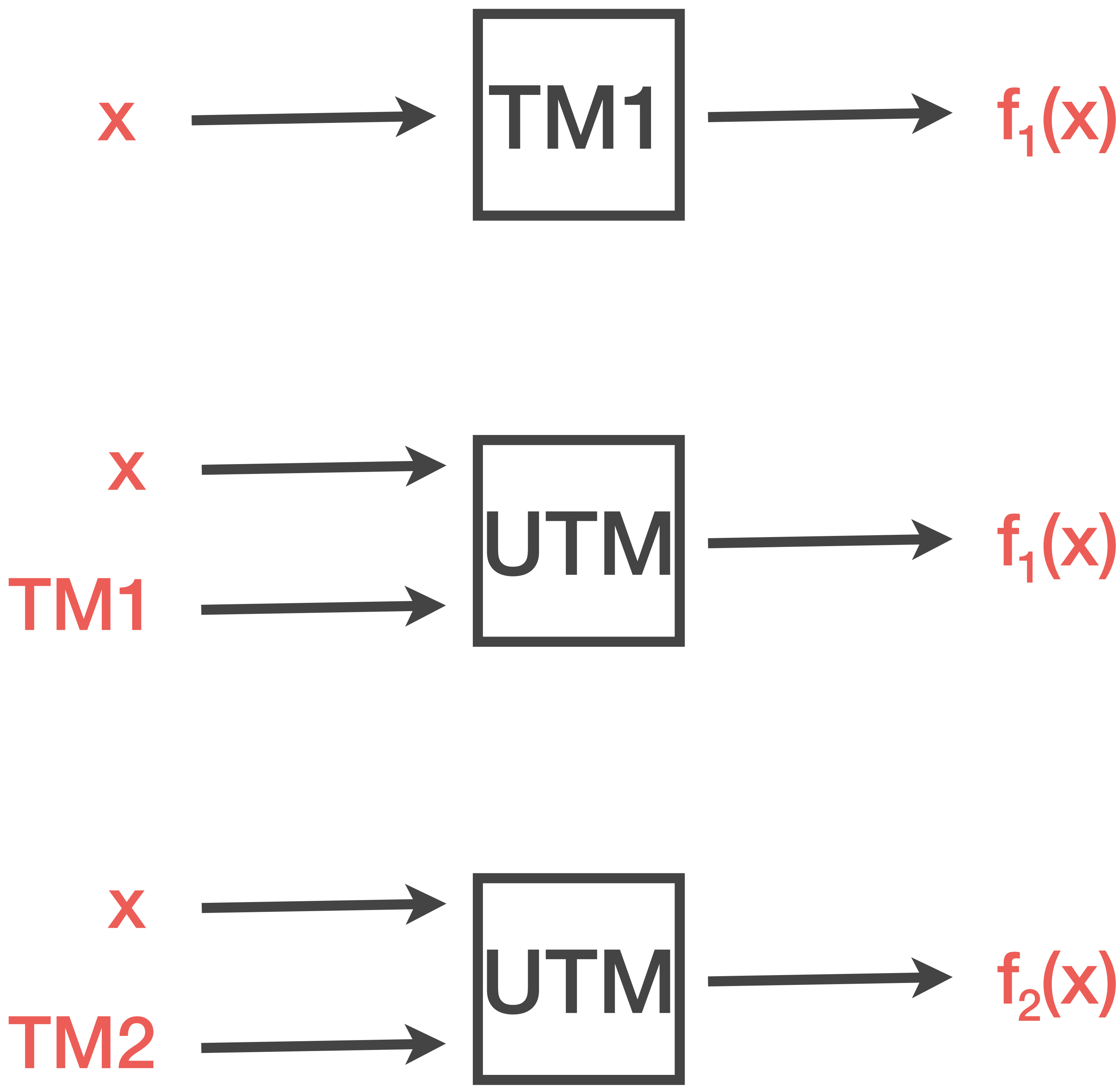


TM as function

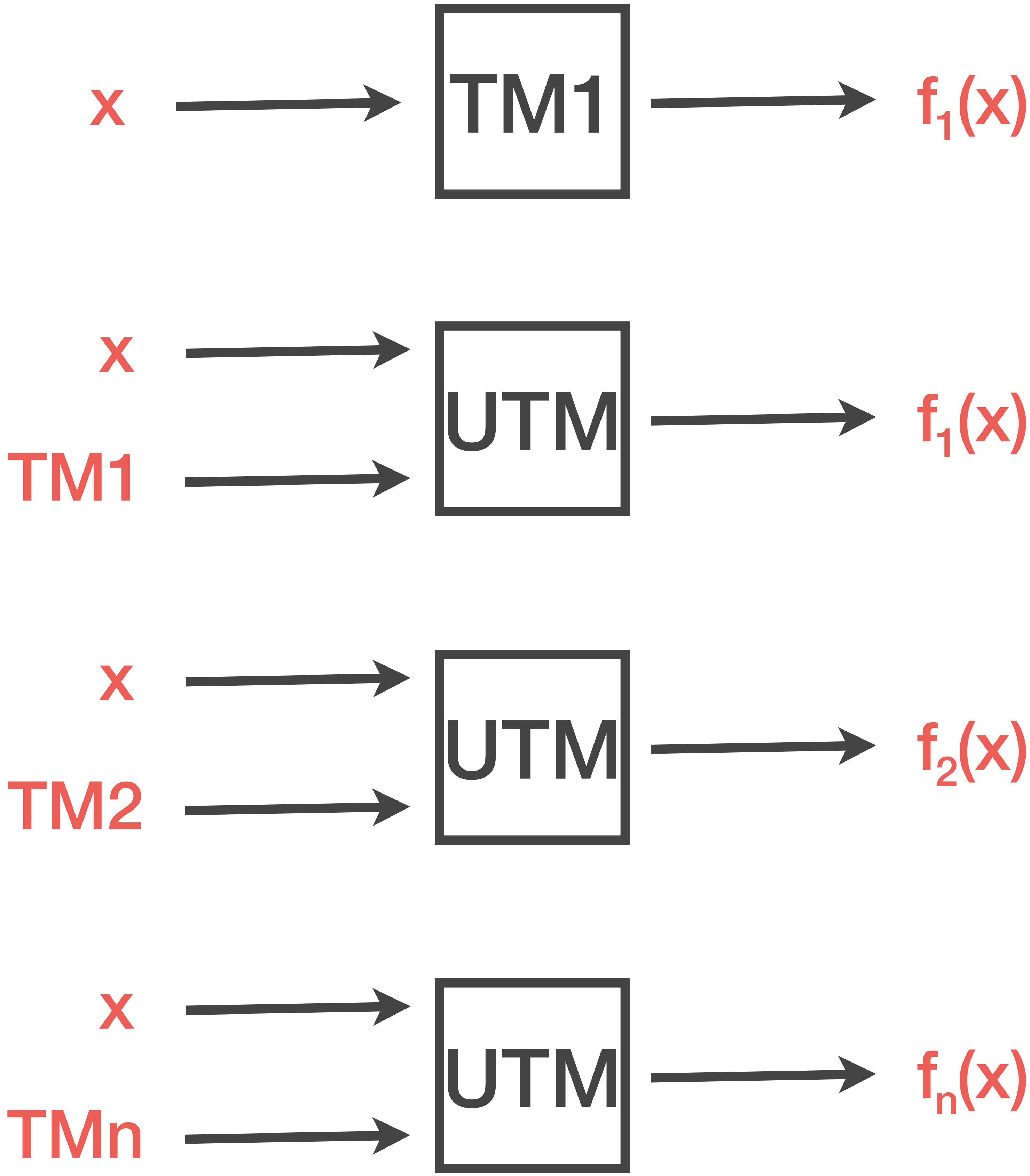



x
TM1
f1(x)


x
TM1
$f_1(x)$
x
TM1
UTM
$f_1(x)$



x
TM1
f1(x)
x
TM1
UTM
f1(x)
x
TM2
UTM
f2(x)



x
TM1
$f_1(x)$
x
TM1
UTM
$f_1(x)$
x
TM2
UTM
$f_2(x)$
x
TMn
UTM
$f_n(x)$

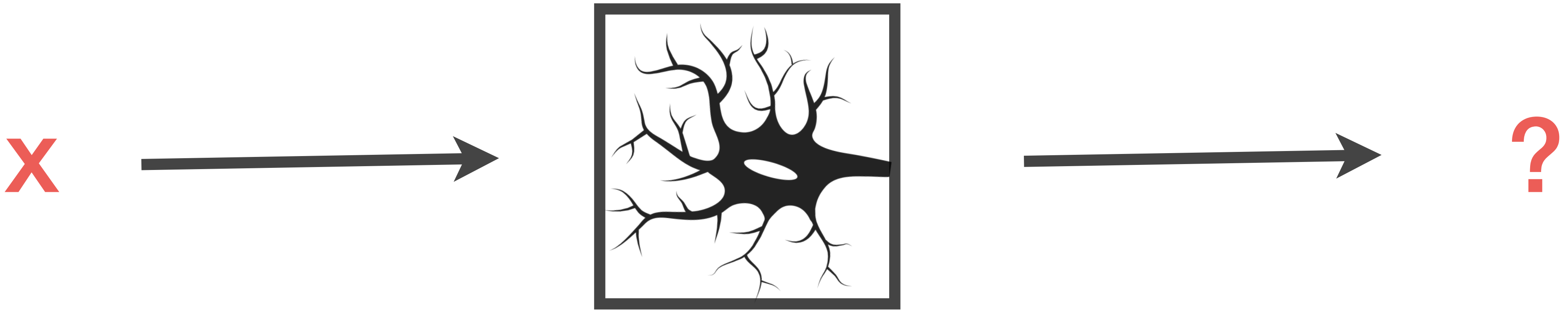
universal TM

There exists a universal Turing machine such that if we give it on tape

- the instructions of another Turing machine
- and its input

then the output will be the output of the given Turing machine given the given input.

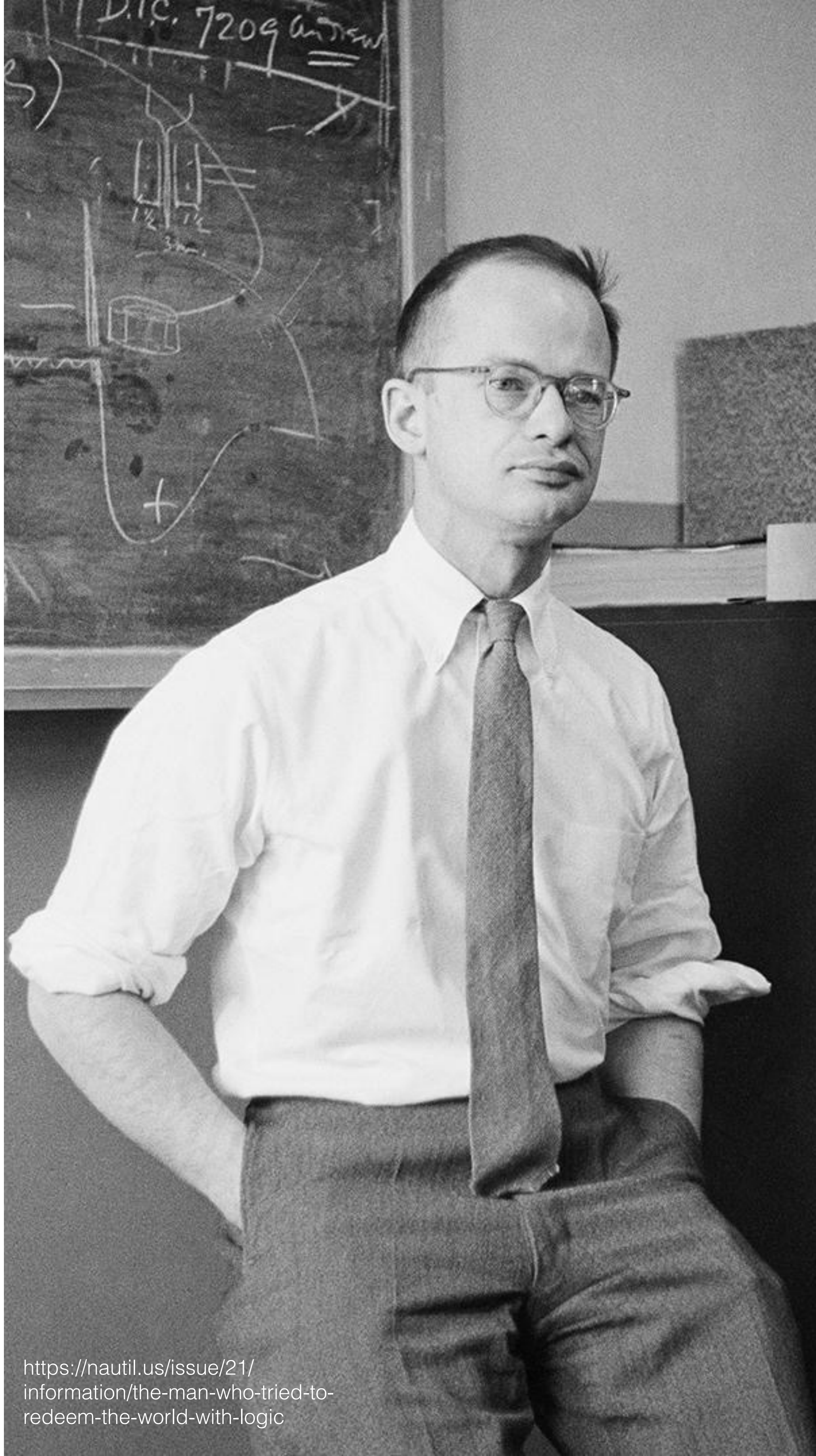
$$\exists F \; : \; F(f_i, x) = f_i(x) \;\; \forall i, x$$



- how does this relate to the brain?
- can the same functions be calculated with neurons?
- with these machines can you calculate everything that the brain can?
    - or maybe neurons know something that nothing else does?
- even something that we cannot model mathematically?

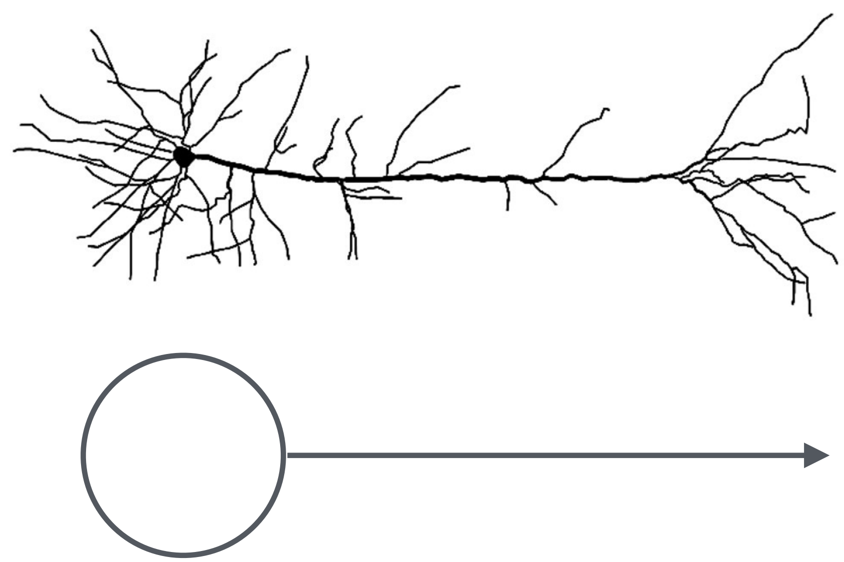
Walter Pitts

1923-1969



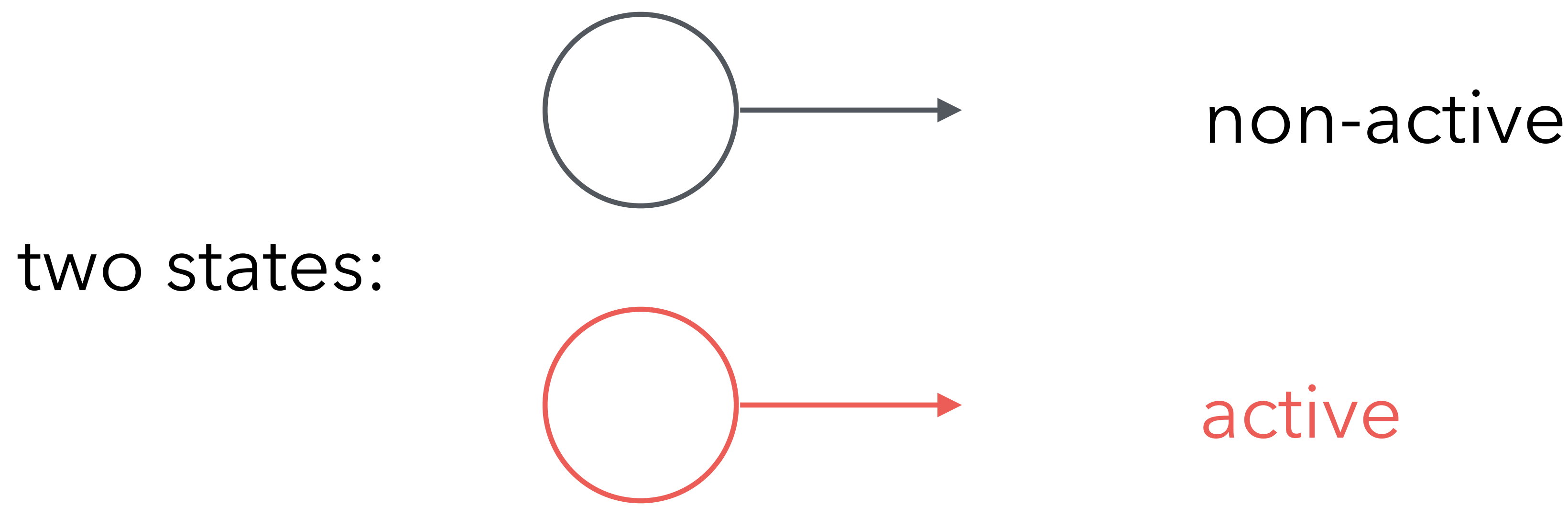
https://nautil.us/issue/21/information/the-man-who-tried-to-redeem-the-world-with-logic

McCulloch-Pitts neuron



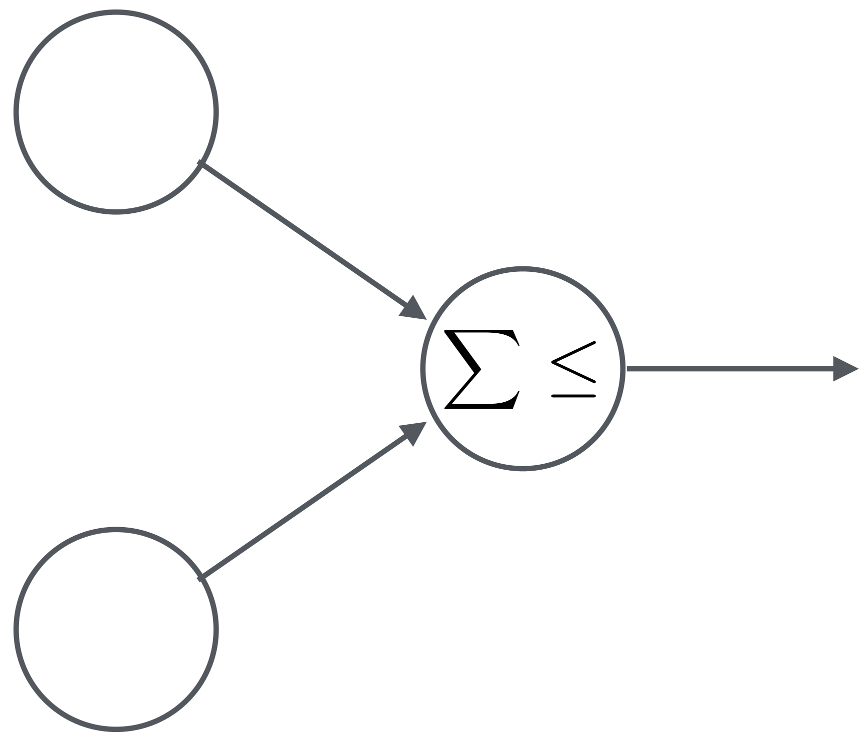
(McCulloch&Pitts 1943)

McCulloch-Pitts neuron



(McCulloch&Pitts 1943)

McCulloch-Pitts neuron



(McCulloch&Pitts 1943)

A Logical Calculus of Ideas Immanent in Nervous Activity

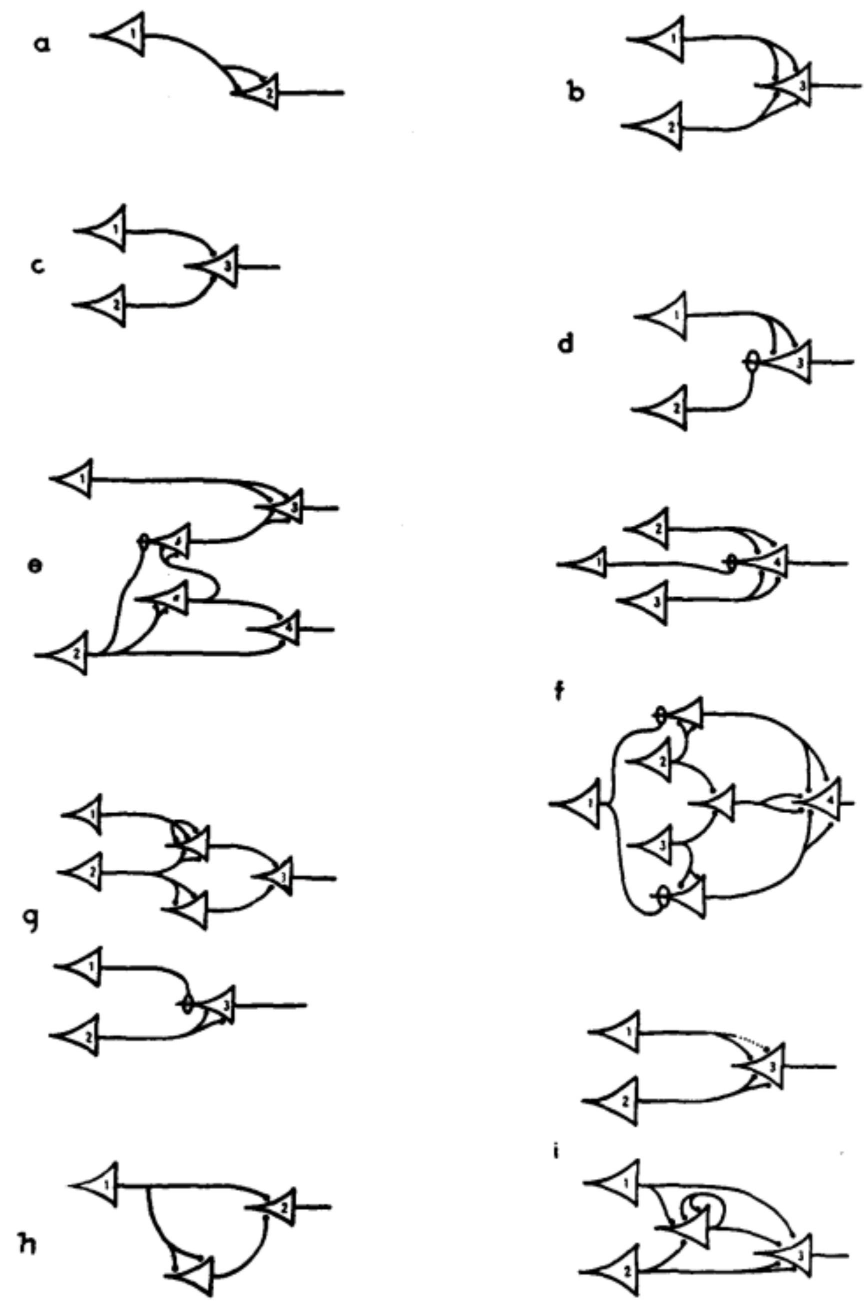
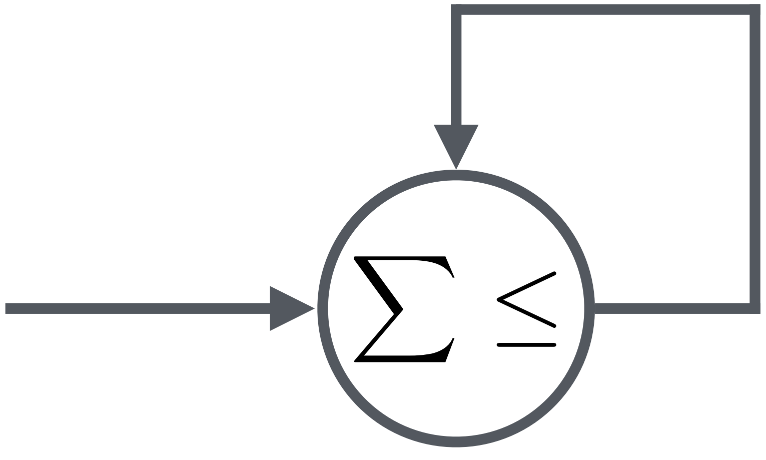
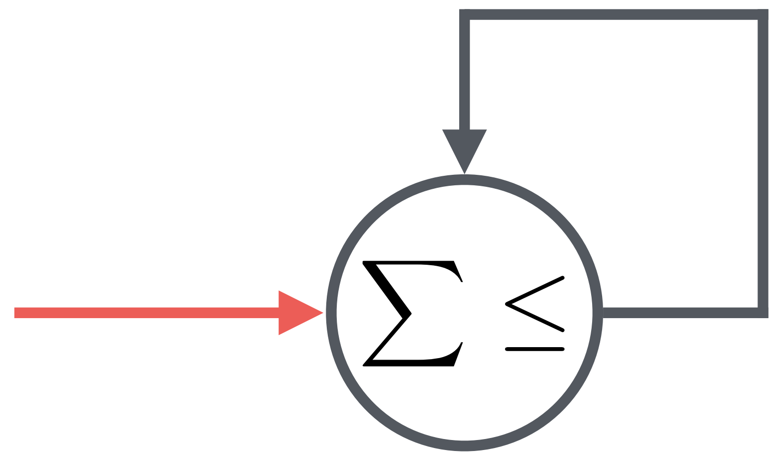


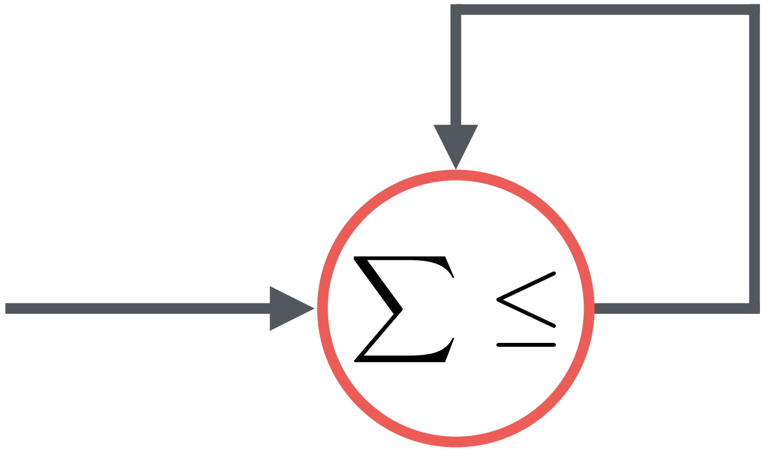
FIGURE 1

(McCulloch&Pitts 1943)

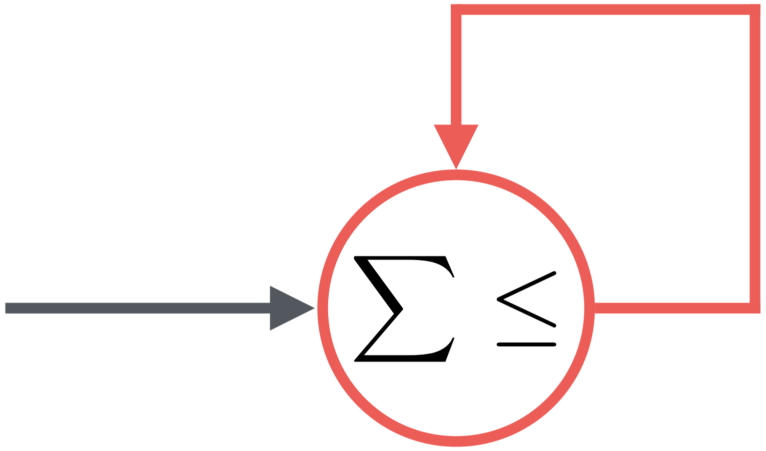
memory?



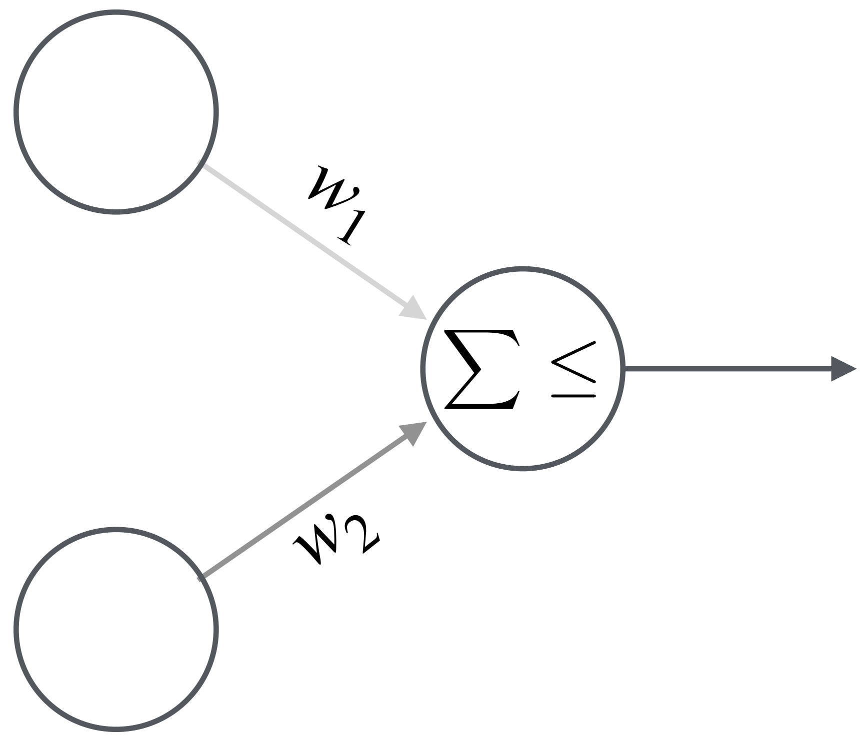

$\sum \leq$



$\sum \leq$

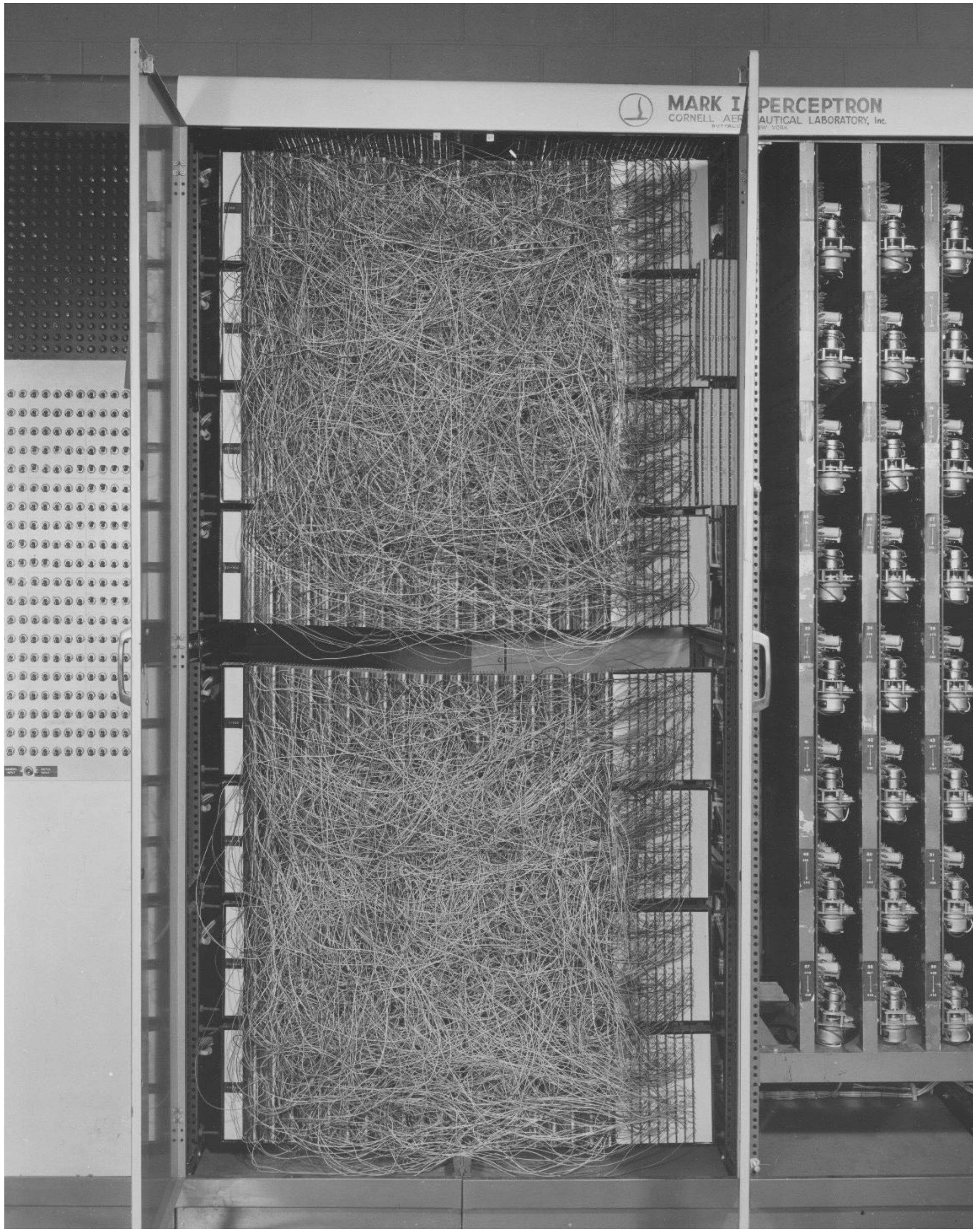

$\sum \leq$

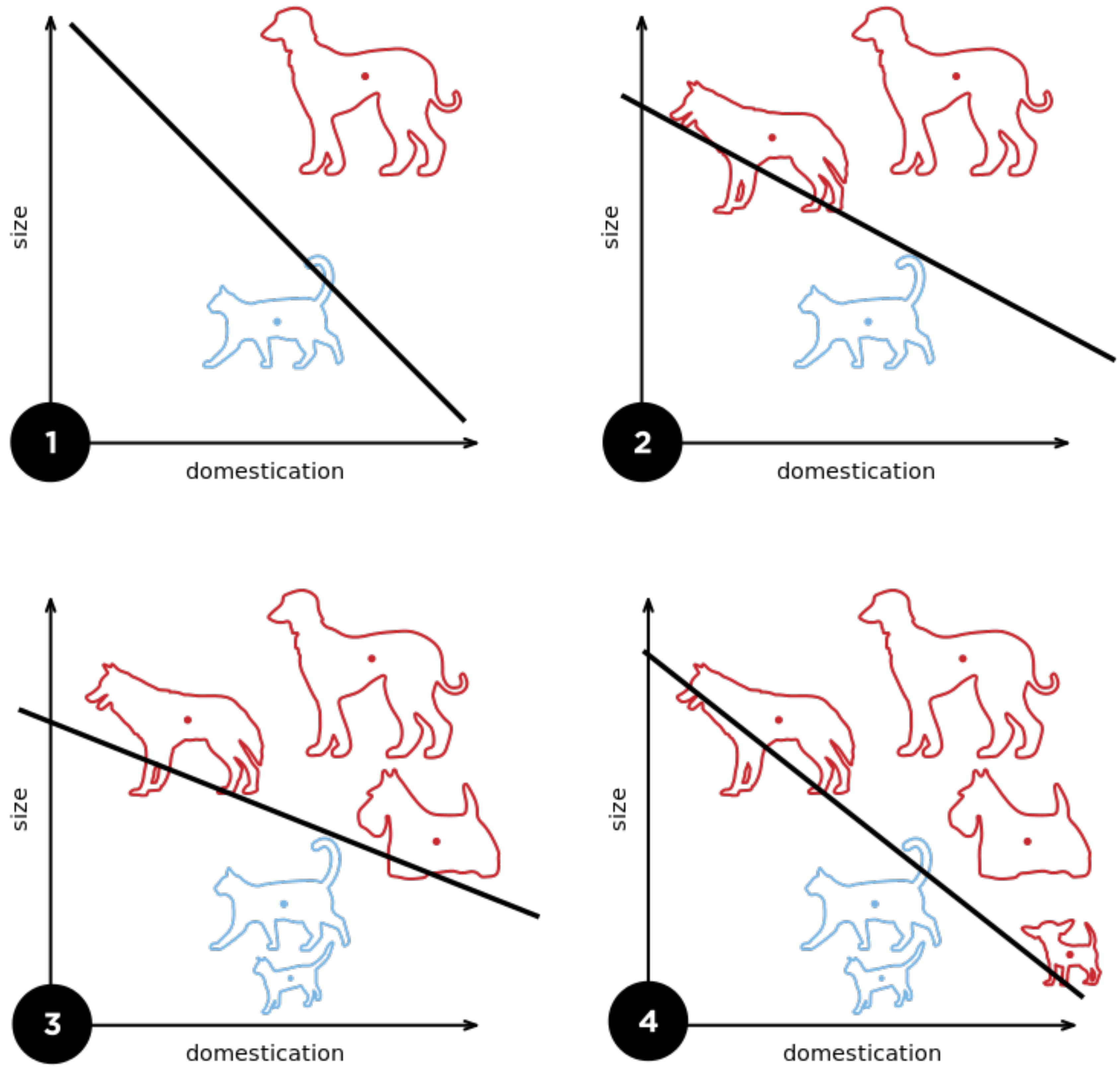
Perceptron

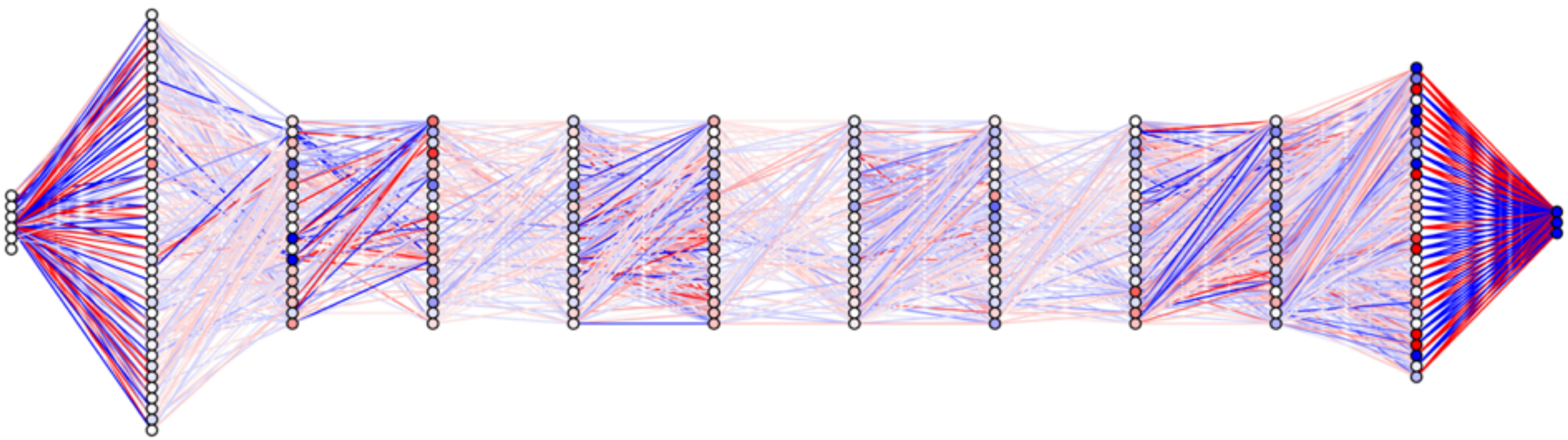


$$\mathbf{W} \circ \mathbf{n}_1 \geq \theta$$

Perceptron
1957

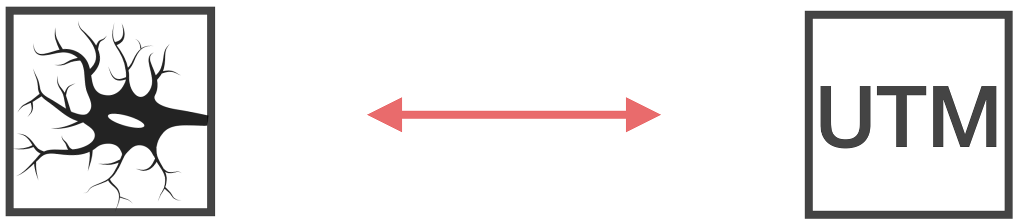


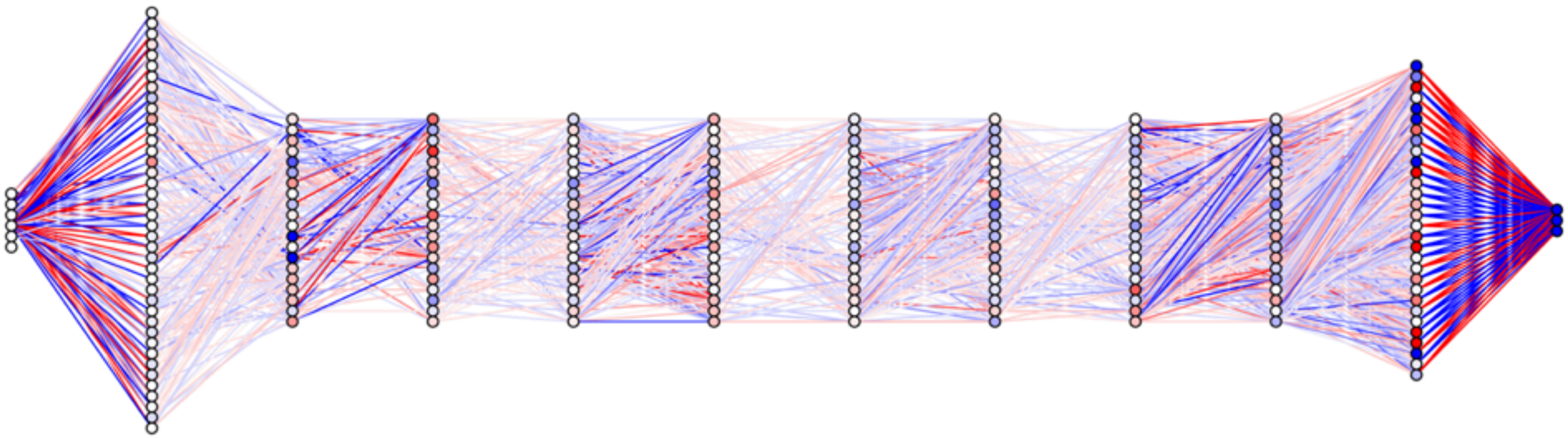

size
domestication
1
size
domestication
2
size
domestication
3
size
domestication
4



$$\mathbf{y} = \sigma \circ \mathbf{W_n} \circ \ldots \circ \sigma \circ \mathbf{W_1} \circ \mathbf{x}$$

Connectionism

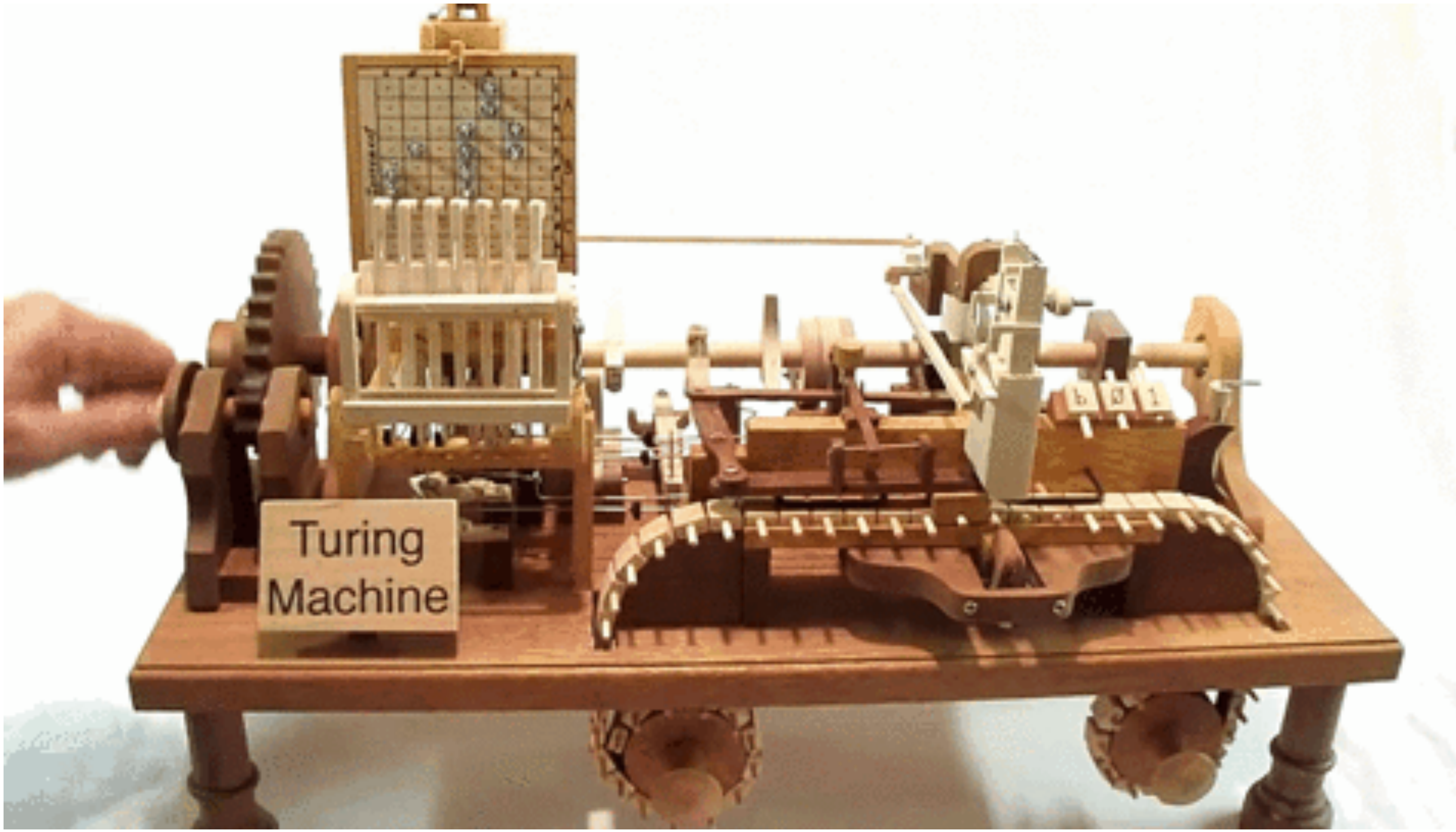


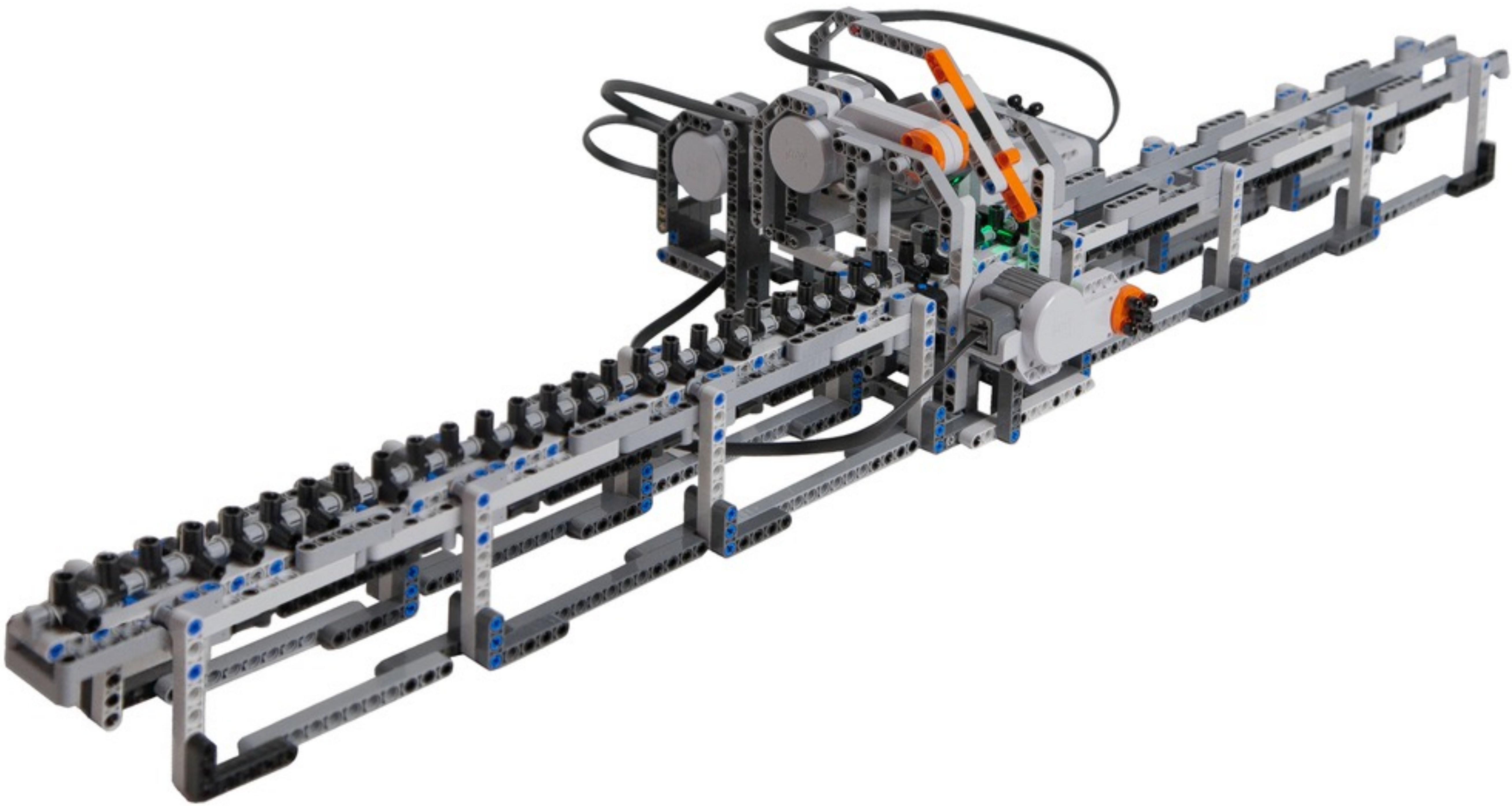


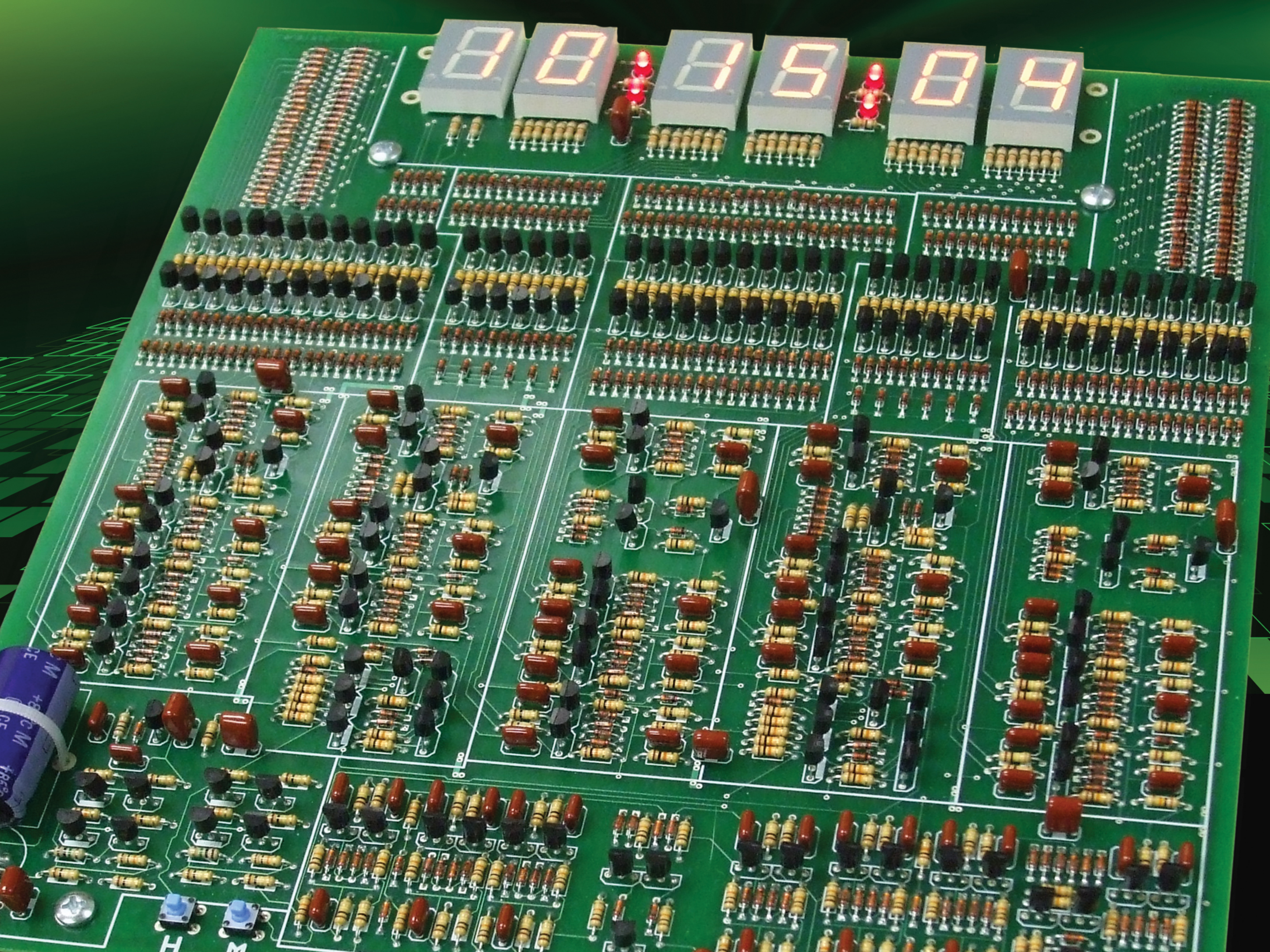
universality of computation

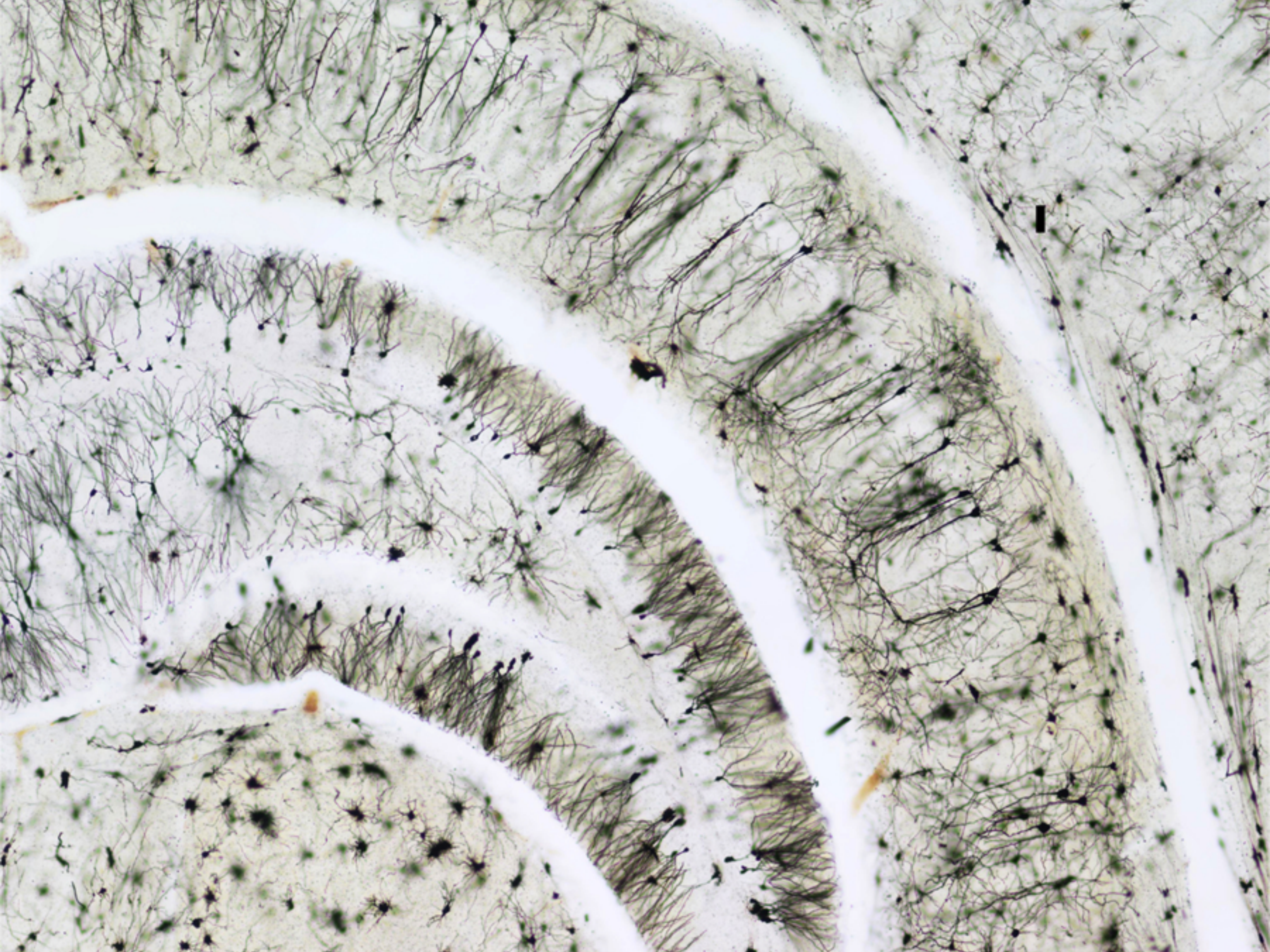
- other general models of computation have also been developed (e.g. recursive functions, λ-calculus)
- surprisingly, it turns out that they define **exactly the same functions** as Turing machines
- **Church-Turing thesis**: anything that can be effectively computed can be computed with UTMs
- these are called **computable functions**
- a stronger claim is that what can be computed by any physical process can be computed by a UTM
    - this is an empirical question, but there are no known counterexamples in physics (called hypercomputation)
    - (e.g. quantum computers can compute the same functions but some of them faster)
- Turing-complete or **universal language**: a set of primitives/components that can be used to create a UTM.

https://plato.stanford.edu/entries/computation-physicalsystems/#AbsComConCom


Turing
Machine




10 15 04
H
M

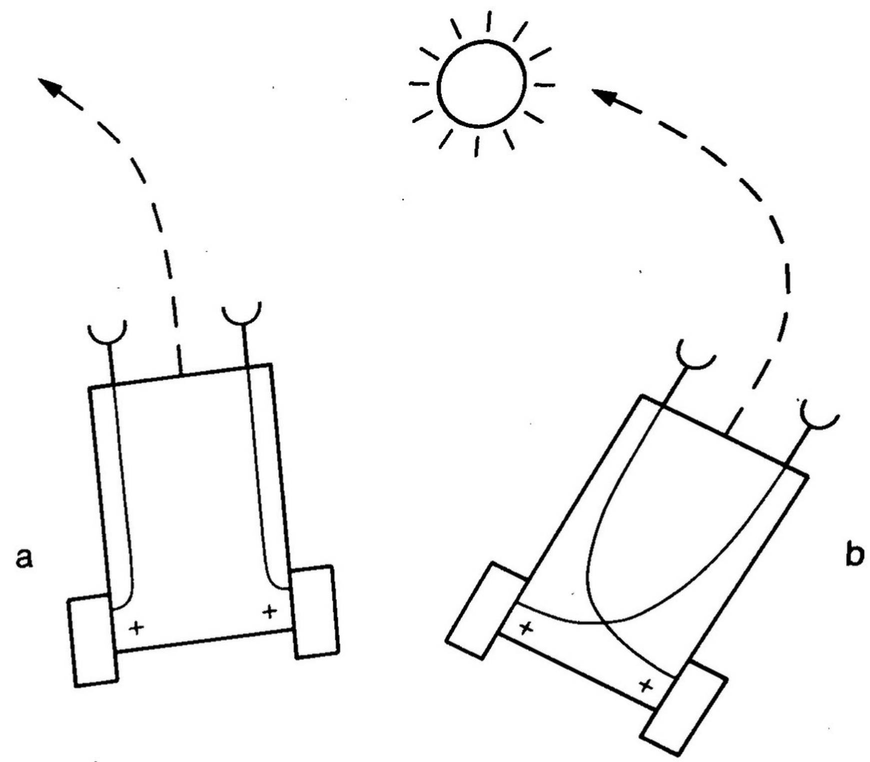


internal model

environment

- model is a simplified “copy” of the original system
    - that can be constructed out of cogs, transistors, or even neurons
    - and in some sense mirrors how the original system works

*: don’t take this too seriously



vs

internal
models

consequences

internal model

environment

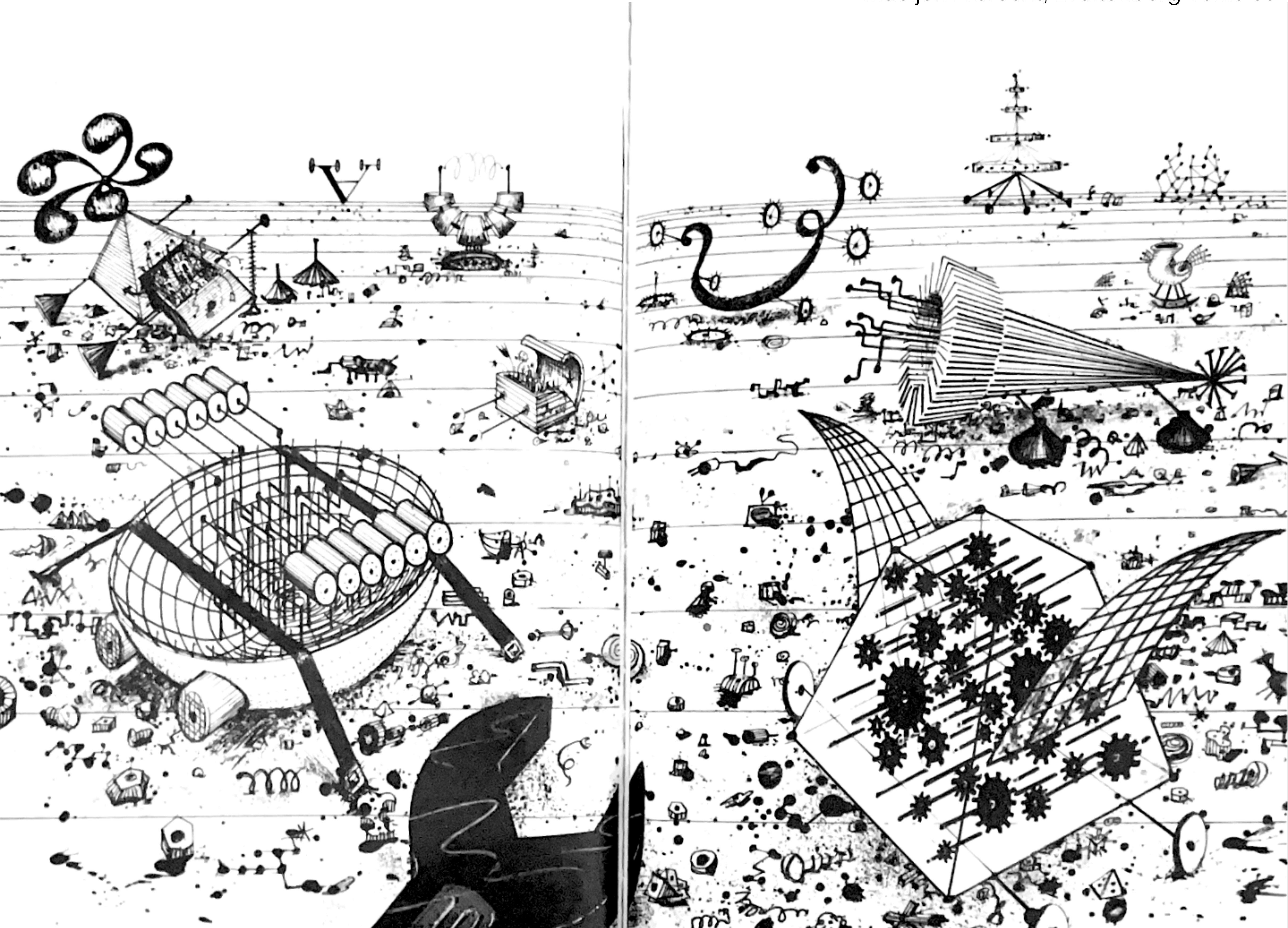
- if we use a Turing-complete modelling language, we will likely be able to model the calculations performed by the brain as well
- the implementation/hardware level can be separated*, we can start modelling from the top down

*: don't take this too seriously

what formalism should we use?

- there are infinitely many, but with increasing complexity they often become equivalent (Turing complete).
  - we can choose one that fits well with the known neurobiology
    - bottom-up approach
    - a potential problem is that our neurobiological knowledge is constantly changing (see McCulloch-Pitts neuron)
    - modeling neurons, neural networks
    - http://cneuro.rmki.kfki.hu/education/neuromodel

Macijek Albrecht, Braitenberg vehicles



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    - a potential problem is that our neurobiological knowledge is constantly changing (see McCulloch-Pitts neuron)
    - modeling neurons, neural networks
    - http://cneuro.rmki.kfki.hu/education/neuromodel
- or we can use a formalism that fits well with the computational problems facing the brain, known behaviour and introspection
    - top-down approach
    - what system makes it simple to formulate and solve representation, perception, learning, reasoning, language use, decision-making, etc.?

logic

All humans are mortal

Aristotle is human

Aristotle is mortal

logic

All greeks wear togas

Socrates is greek

Socrates wears a toga

logic

All **A** are **B**

X is an **A**

Therefore **X** is **B**

syllogism

logical system

- what are distinct states of the system (propositions)?
    - we decompose these using state variables (atomic propositions)
- which of these states are possible states of the system?
    - we can summarise these in a **truth table**

truth table

atomic propositions (variables)

cough	has cold	has TB

truth table

atomic propositions (variables)

cough	has cold	has TB
1	1	1
1	1	0
1	0	1
1	0	0
0	1	1
0	1	0
0	0	1
0	0	0

distinct states of system

truth table

how can we use the table for drawing inferences?

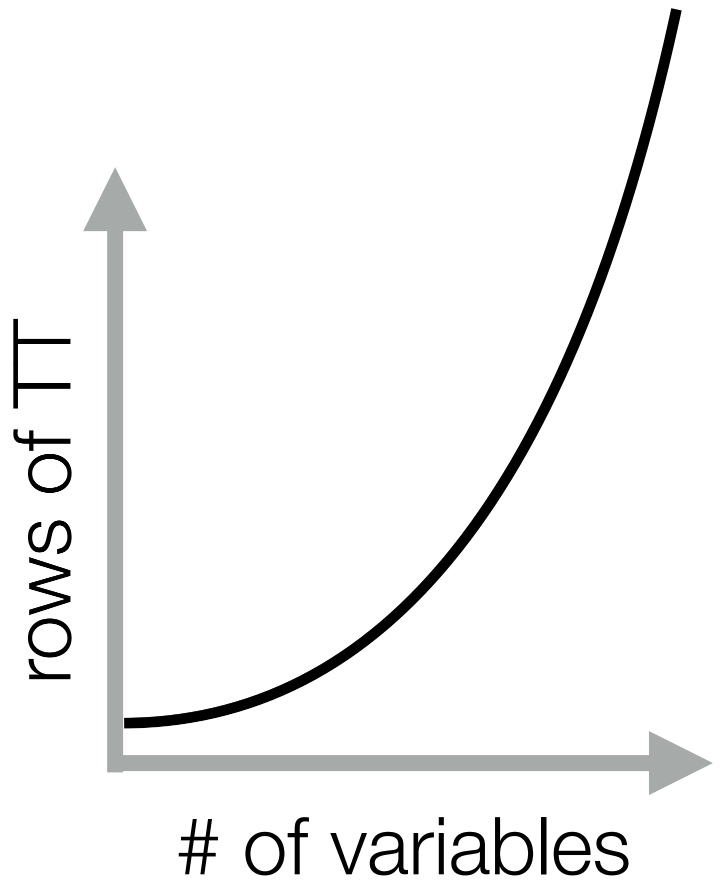
cough	has cold	has TB	possible?
1	1	1	1
1	1	0	1
1	0	1	1
1	0	0	0
0	1	1	0
0	1	0	0
0	0	1	0
0	0	0	1

inference

if sj. coughs but not cold, is it TB? yes

cough	has cold	has TB	possible?	
~~1~~	~~1~~	~~1~~	~~1~~	has cold
~~1~~	~~1~~	~~0~~	~~1~~	
1	0	1	1	
~~1~~	~~0~~	~~0~~	~~0~~	
~~0~~	~~1~~	~~1~~	~~0~~	no cough
~~0~~	~~1~~	~~0~~	~~0~~	
~~0~~	~~0~~	~~1~~	~~0~~	
~~0~~	~~0~~	~~0~~	~~1~~	

1st problem



megoldás

introduce operators to more compactly represent truth table

$$\neg, \ \vee, \ \wedge, \ \rightarrow, \ \leftrightarrow$$

A	B	A ∨ B
t	t	t
t	f	t
f	t	t
f	f	f

operators

$\neg$	negation
$\vee$	or
$\wedge$	and
$\rightarrow$	if, then
$\leftrightarrow$	if and only if

conditional truth table

A	B	A $\vee$ B
t	t	t
t	f	t
f	t	t
f	f	f

operators

cough	has cold	has TB	possible?
1	1	1	1
1	1	0	1
1	0	1	1
1	0	0	0
0	1	1	0
0	1	0	0
0	0	1	0
0	0	0	1

$$(M \vee T) \leftrightarrow K$$

new rules of inference

$$\neg\neg A \rightarrow A$$

if it is not true that it is not A then A

$$A, B \rightarrow A \wedge B$$

if it is true that A and it is true that B then it is true that A and B

$$(\neg A \wedge \neg B) \rightarrow \neg(A \vee B)$$

if it is not A and not B, then it is not true that A or B

$$((A \rightarrow B) \wedge A) \rightarrow B$$

if given A it is true that B, and A is true, then B is true

higher order logics

boole operators

truth value to truth value

$\neg, \ \vee, \ \wedge, \ \rightarrow$

A	B	$A \vee B$
t	t	t
t	f	t
f	t	t
f	f	f

truth table

propositional logic

higher order logics

truth table

propositional logic

1st order logic

predicates
objects to truth values

$$Fekete(kutya)$$

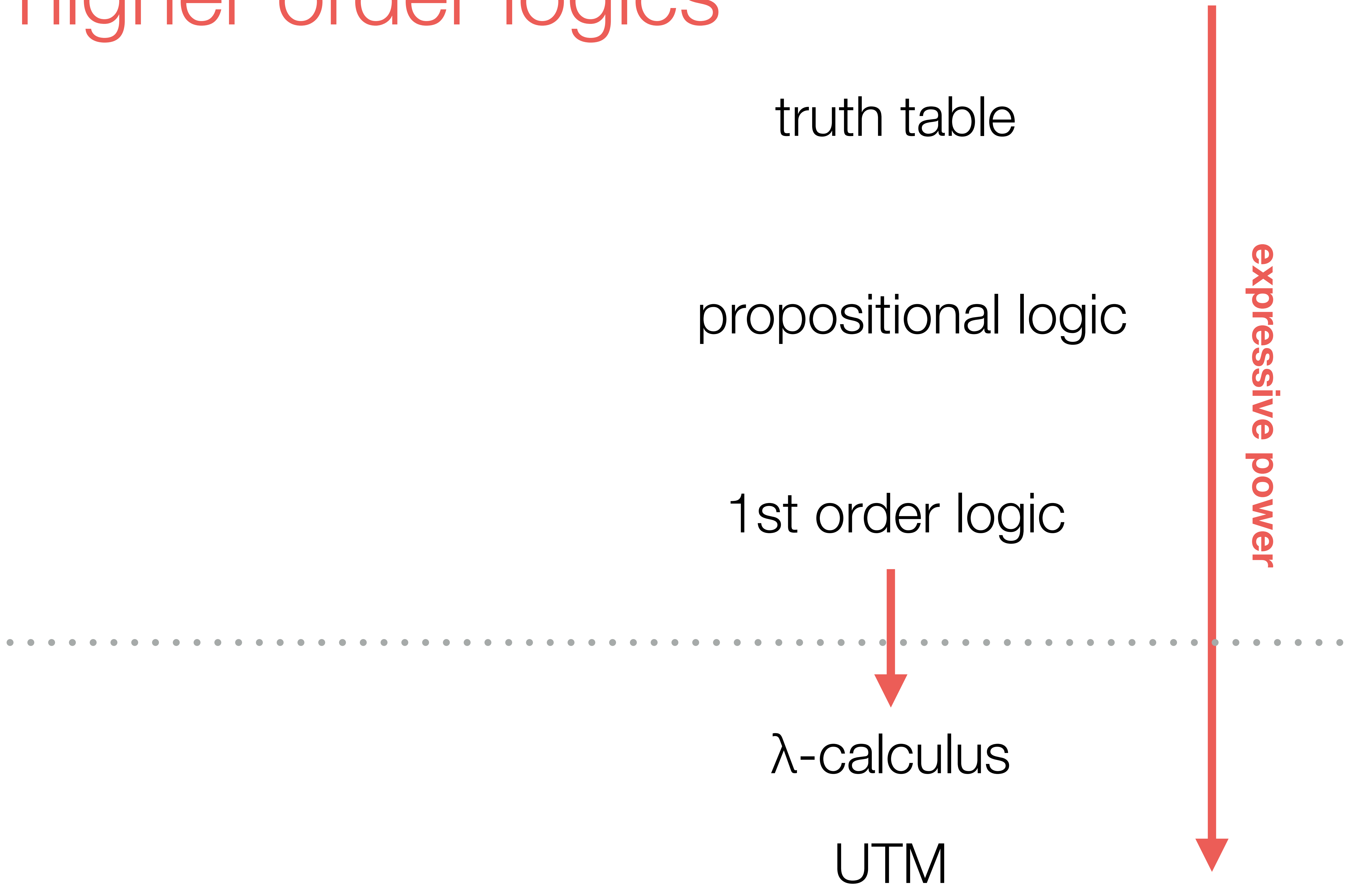
quantifiers
object variables to objects

$$\forall, \exists$$

rules of chess
propositional logic ~ 1000 pages
first order logic ~ 1 page

expressive power

higher order logics



2nd problem

if subject coughs, is it TB?

¯_(ツ)_/¯

cough	has cold	has TB	possible?
1	1	1	1
1	1	0	1
1	0	1	1
1	0	0	0
0	1	1	0
0	1	0	0
0	0	1	0
0	0	0	1

$\in \mathbb{R}$

how should we choose these numbers?

Cox-theorem

- represent plausibility (H) with real numbers
- consistency: regardless of which order we apply rules to compute plausibilities, we should get the same number given the same information
- direction of changes of plausibility

if $H(A|i)$ increases, then

$H(\neg A|i)$ decreases

$H(A \wedge B|i)$ increases

- If plausibility satisfies these requirements then it is isomorphic to a probability measure, with the usual definition of conditional probability

$$H(\cdot|i) \simeq P(\cdot|i)$$

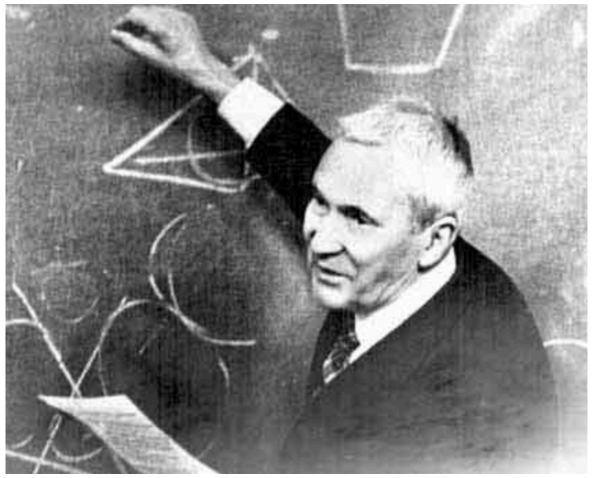
Dutch Book argument

- assumes that we are willing to make bets according to our degrees of belief/plausilities
- if our beliefs don't satisfy these consistency rules, then there will exist a set of bets that the we would accept, even though it would guarantee a loss

goals for today

- what is knowledge?
- what are internal models?
- what are the limits of representing knowledge with neurons?
- **why are we going to use probability theory as a mathematical framework in the rest of the course?**

Kolmogorov axioms

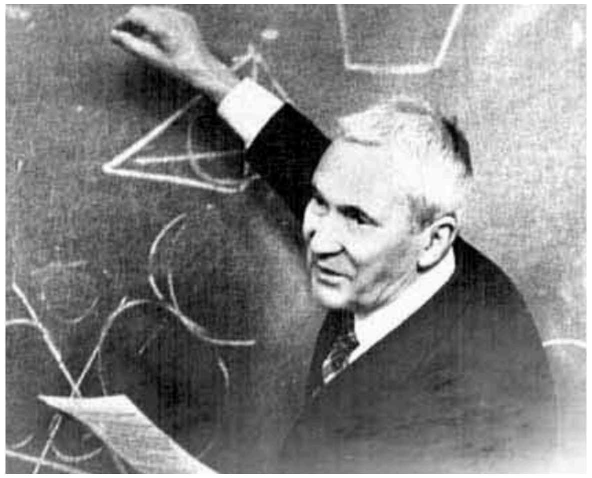


- probabilities of events are real numbers btwn. 0 and 1

cough	sneeze	flu	TB	probability
t	t	t	t	0.1
t	t	f	f	0.02
...	...	...	...	...
f	f	f	f	0.3

$\in (0, 1)$

Kolmogorov axioms



- probabilities of events are real numbers btwn. 0 and 1
- probability of definite event is 1
- probabilities of mutually exclusive events are additive

cough	sneeze	flu	TB	probability
t	t	t	t	0.1
t	t	f	f	0.02
...	...	...	...	...
f	f	f	f	0.3

$$\sum P(w_i) = 1$$

inference = conditional prob.

$$P(TB|cough, \neg flu) = ?$$

cough	sneeze	flu	TB	probability
~~t~~	~~t~~	~~t~~	~~t~~	~~0.1~~
t	t	f	f	0.02
...	...	...	...	...
~~f~~	~~f~~	~~f~~	~~f~~	~~0.3~~

flu

no cough

rest must sum to 1

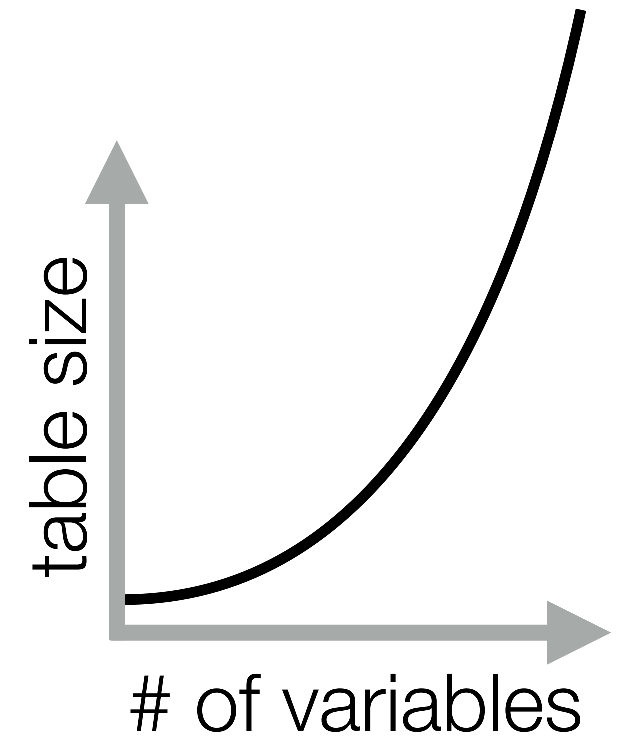
- "what is the probability of A given B?"

$$P(A|B) = \frac{P(A, B)}{P(B)}$$

- can be seen as generalisation of $\longrightarrow$

problem

- $2^n - 1$ numbers are needed to specify full probability table
- could we use same trick as before?
    - compose full table from smaller tables?



- probability distribution can be written as product of conditionals

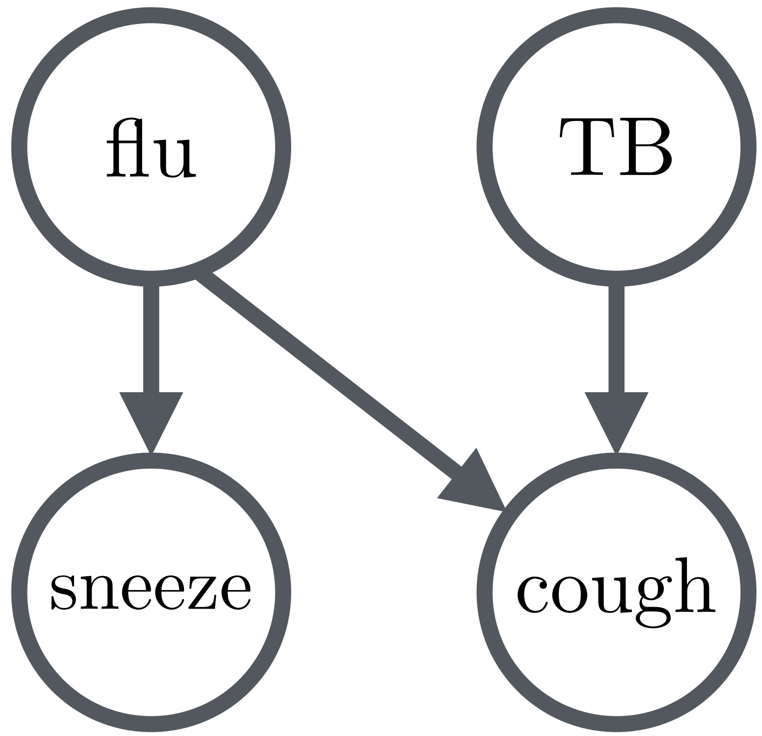
$$P(x_1, x_2, x_3, ..., x_n) = P(x_1|x_2)P(x_2|x_3)...P(x_n)$$

- this can be used to introduce graphical models
    - we will look at a simple example now, and go into more detail during the next lecture

graphical model

flu=t
0.2

TB=t
0.1



flu	sneeze=t
t	0.8
f	0.2

TB	flu	cough=t
t	t	0.9
t	f	0.8
f	t	0.75
f	f	0.1

logic		**probability**
truth table	$\longleftrightarrow$	joint probability table
propositional logic	$\longleftrightarrow$	d. graphical model
1st order logic		
λ-calculus	$\longleftrightarrow$	ψλ -calculus
UTM	$\longleftrightarrow$	UPTM

logic	**probability**
truth table	joint probability table
propositional logic	d. graphical model
1st order logic	
λ-calculus UTM	probabilistic programs

probabilistic program

- Turing-complete language

- random choice operator

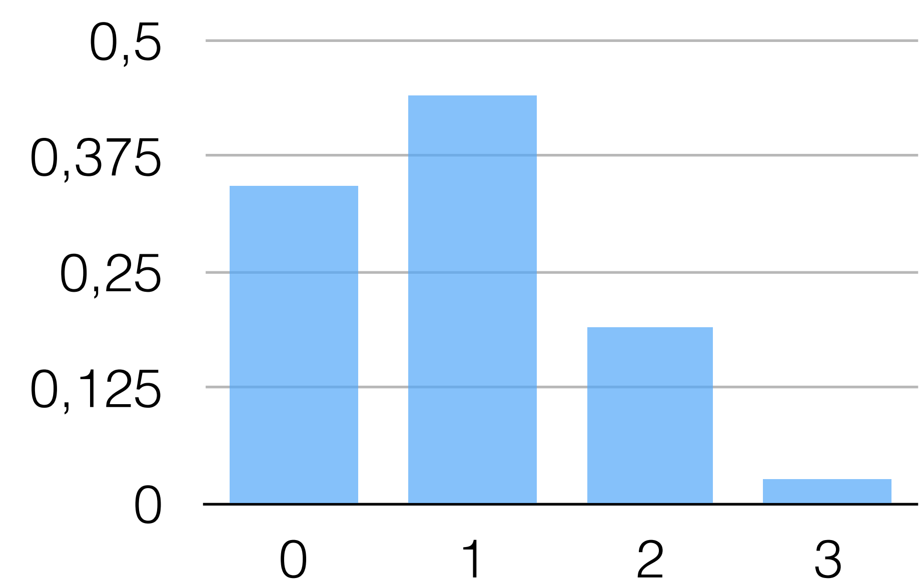
```
flip (0.5)
```

0 or 1

- conditioning (inference) as language primitive

probabilistic program

`a = flip(0.3)`	1 0 0
`b = flip(0.3)`	0 0 0
`c = flip(0.3)`	1 0 1
`a + b + c`	2 0 1



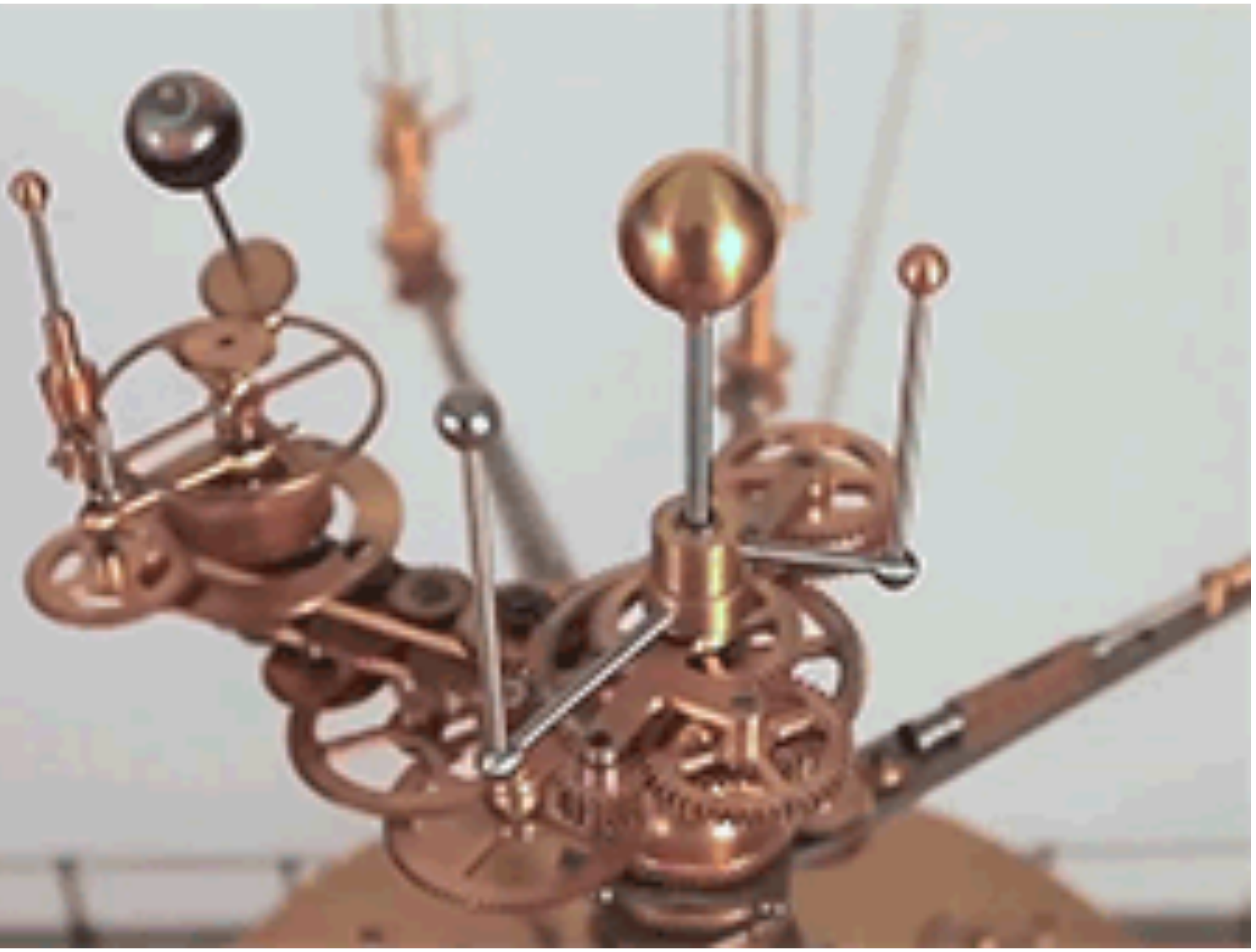
$$P(n) = \binom{3}{n} 0.3^n 0.7^{3-n}$$

sampling

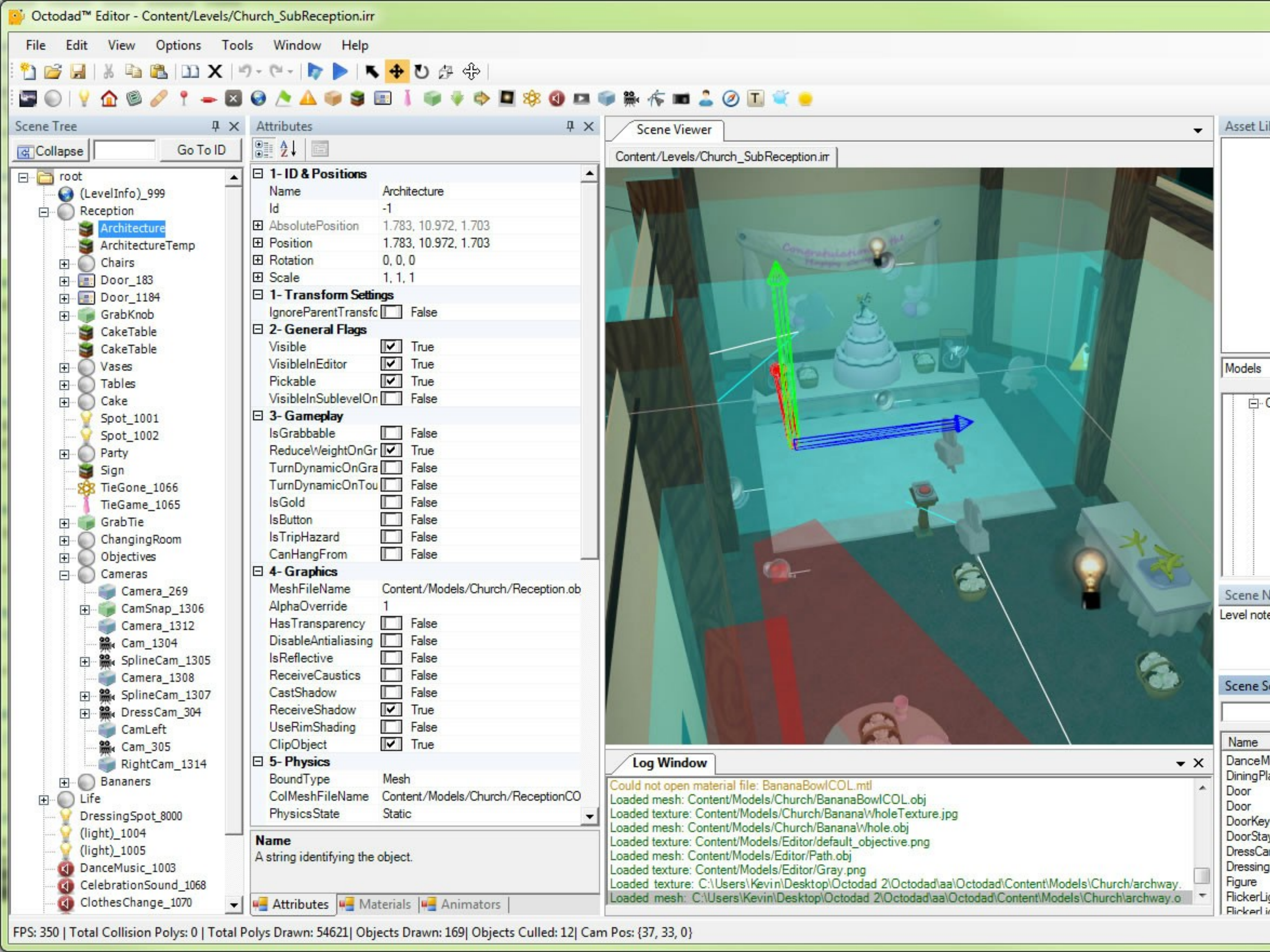
distribution

Pyro

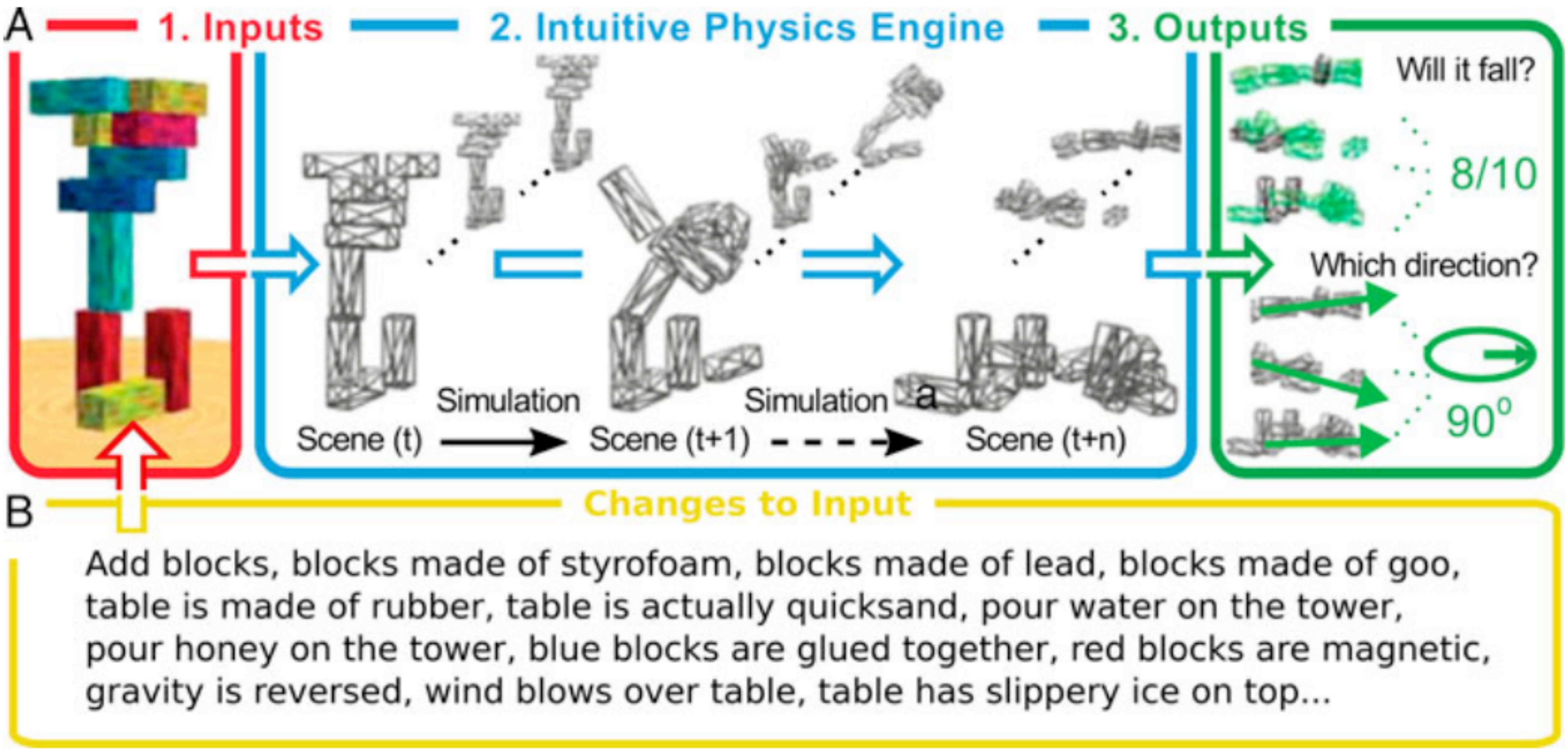
Edward



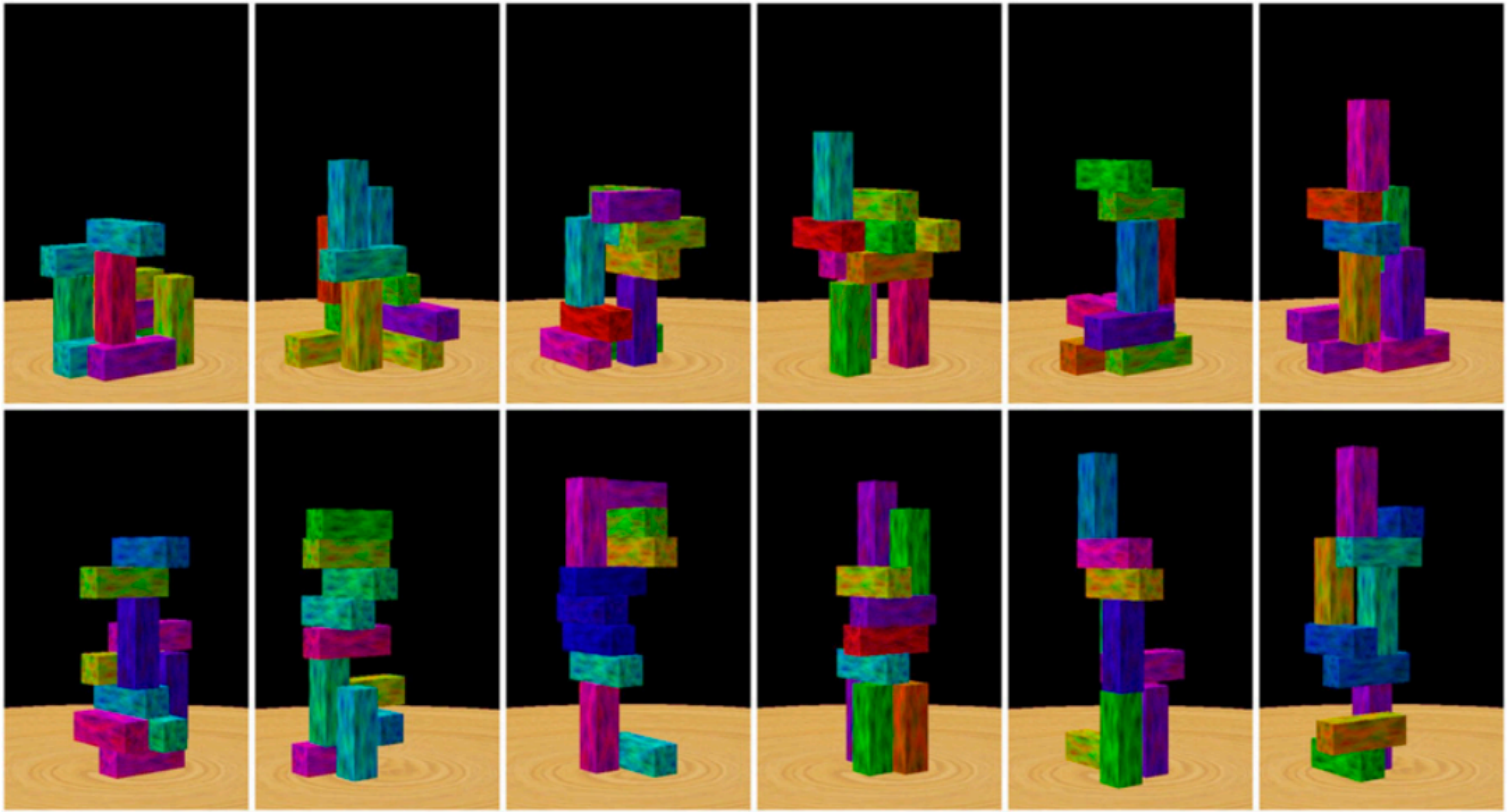
probabilistic model
(program/distribution)


Octodad™ Editor - Content/Levels/Church_SubReception.irr
File
Edit
View
Options
Tools
Window
Help
Scene Tree
Collapse
Go To ID
root
(LevelInfo)_999
Reception
Architecture
ArchitectureTemp
Chairs
Door_183
Door_1184
GrabKnob
CakeTable
CakeTable
Vases
Tables
Cake
Spot_1001
Spot_1002
Party
Sign
TieGone_1066
TieGame_1065
GrabTie
ChangingRoom
Objectives
Cameras
Camera_269
CamSnap_1306
Camera_1312
Cam_1304
SplineCam_1305
Camera_1308
SplineCam_1307
DressCam_304
CamLeft
Cam_305
RightCam_1314
Bananers
Life
DressingSpot_8000
(light)_1004
(light)_1005
DanceMusic_1003
CelebrationSound_1068
ClothesChange_1070
Attributes
1- ID & Positions
Name Architecture
Id -1
AbsolutePosition 1.783, 10.972, 1.703
Position 1.783, 10.972, 1.703
Rotation 0, 0, 0
Scale 1, 1, 1
1- Transform Settings
IgnoreParentTransfc False
2- General Flags
Visible True
VisibleInEditor True
Pickable True
VisibleInSublevelOn False
3- Gameplay
IsGrabbable False
ReduceWeightOnGr True
TurnDynamicOnGra False
TurnDynamicOnTou False
IsGold False
IsButton False
IsTripHazard False
CanHangFrom False
4- Graphics
MeshFileName Content/Models/Church/Reception.ob
AlphaOverride 1
HasTransparency False
DisableAntialiasing False
IsReflective False
ReceiveCaustics False
CastShadow False
ReceiveShadow True
UseRimShading False
ClipObject True
5- Physics
BoundType Mesh
ColMeshFileName Content/Models/Church/ReceptionCO
PhysicsState Static
Name
A string identifying the object.
Attributes
Materials
Animators
Scene Viewer
Content/Levels/Church_SubReception.irr
Log Window
Could not open material file: BananaBowlCOL.mtl
Loaded mesh: Content/Models/Church/BananaBowlCOL.obj
Loaded texture: Content/Models/Church/BananaWholeTexture.jpg
Loaded mesh: Content/Models/Church/BananaWhole.obj
Loaded texture: Content/Models/Editor/default_objective.png
Loaded mesh: Content/Models/Editor/Path.obj
Loaded texture: Content/Models/Editor/Gray.png
Loaded texture: C:\Users\Kevin\Desktop\Octodad 2\Octodad\aa\Octodad\Content\Models\Church\archway.
Loaded mesh: C:\Users\Kevin\Desktop\Octodad 2\Octodad\aa\Octodad\Content\Models\Church\archway.o
Models
Level note
Name
FPS: 350 | Total Collision Polys: 0 | Total Polys Drawn: 54621| Objects Drawn: 169| Objects Culled: 12| Cam Pos: (37, 33, 0)

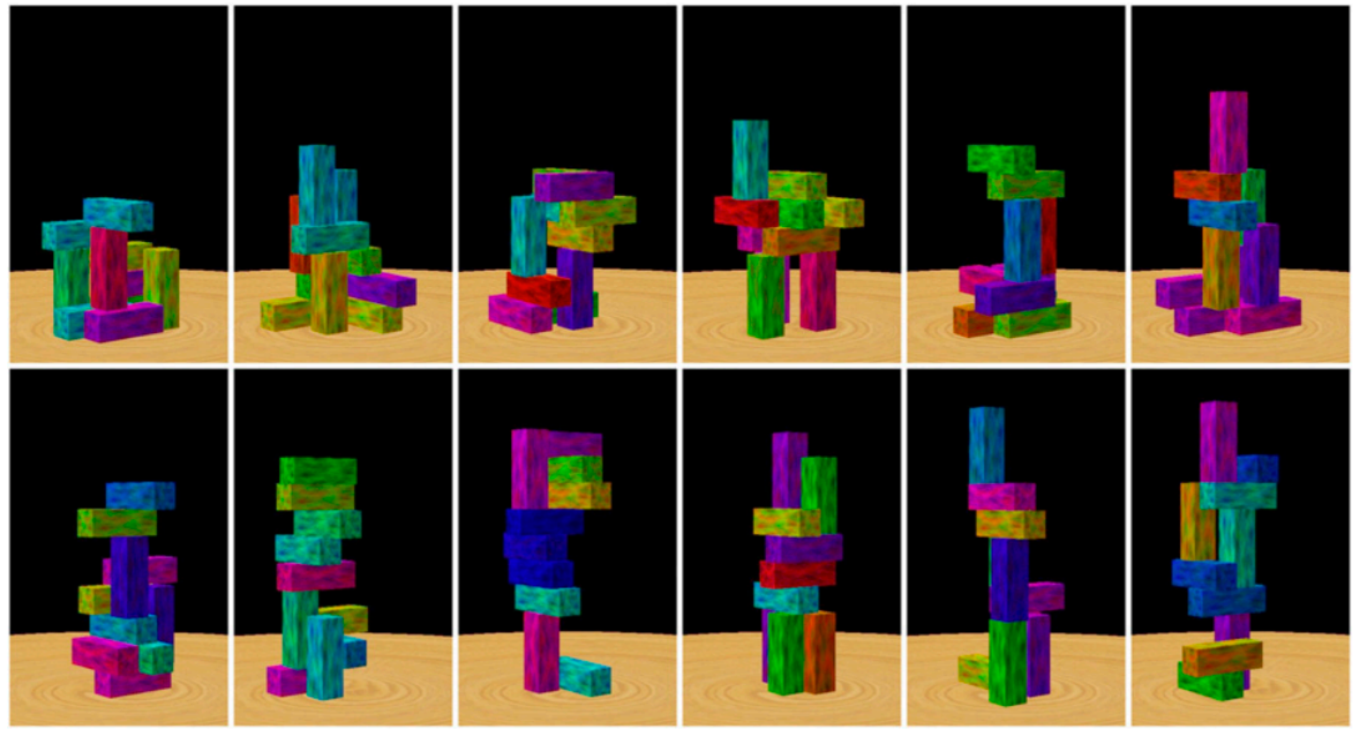
File
Edit
View
Editors
Camera
Renderer
Engine
Tools
DEV
Help
Scene Entity List
Lights
Default - Light Directional
Characters
Knightress_153
Knightress_154
Knightress_152
Knightress_151
Knightress_154
Scene
City Wall
Fence
Misc
Rocks
Rock2_2
Rock1B_247
Rock4A_12
Rock2_46
Rock2_29
Rock2_75
Rock2_40
Rock2_8
Rock4A_120
Rock2_6
Rock2_26
Skyline Console
Finished Updating Billboards
Finished Updating Billboards
Finished Updating Billboards
LOADING XML: Terrain Road Settings
LOADING XML: CAMERA Settings
LOADING XML: Load Bookmarks
LOADING XML: Post Effects
LOADING XML: SHADOW Settings
LOADING XML: Navmesh Settings
******* XML :: MISSING FILE:
| LOAD LEVEL COMPLETED [XML]
Clear Log
Enter Commands Here....
World Space
1.00
Terrain Editor
Creation
Terrain Generator
Terrain Setup
Terrain Size (Resolution)
512
World Size (Size in World Units
1000
Texture Blend Map Size:
1024
Tiles X
0
Tiles Y
0
Generate
Terrain Options:
Generate
Remove
Apply Physics
Import Options:
Export Options:
Load Terrain file
Export Terrain file
Load HeightMap
Export HeightMap
Terrain Properties
Heightmap Multiplier
75
Use Surface Names
Update Surfaces
Open AM Heightmaps
Open
Lightmap
Material
Brush Size
Brush Strength
Terrain Editor

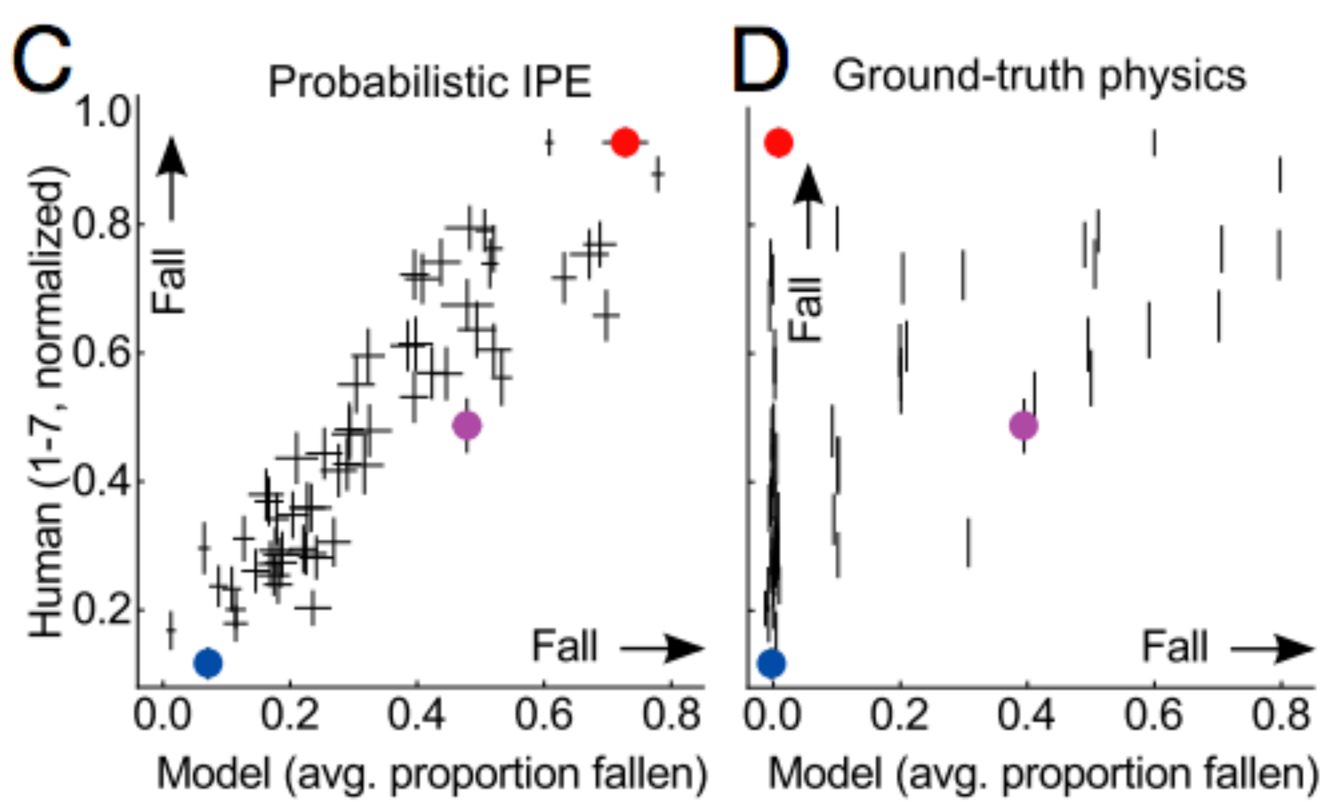


[Lake et al, 2016]



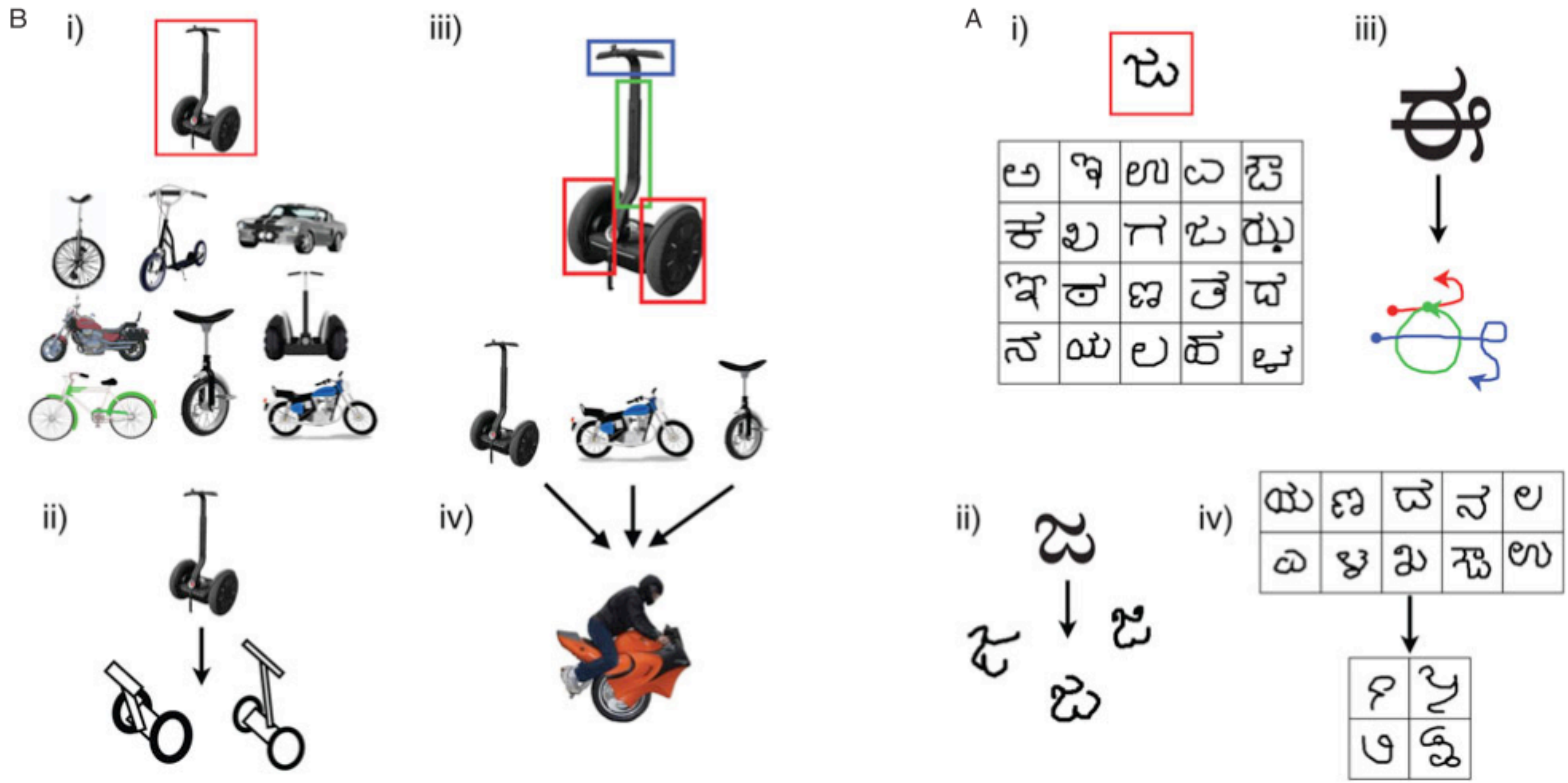
[Battaglia et al., 2013]



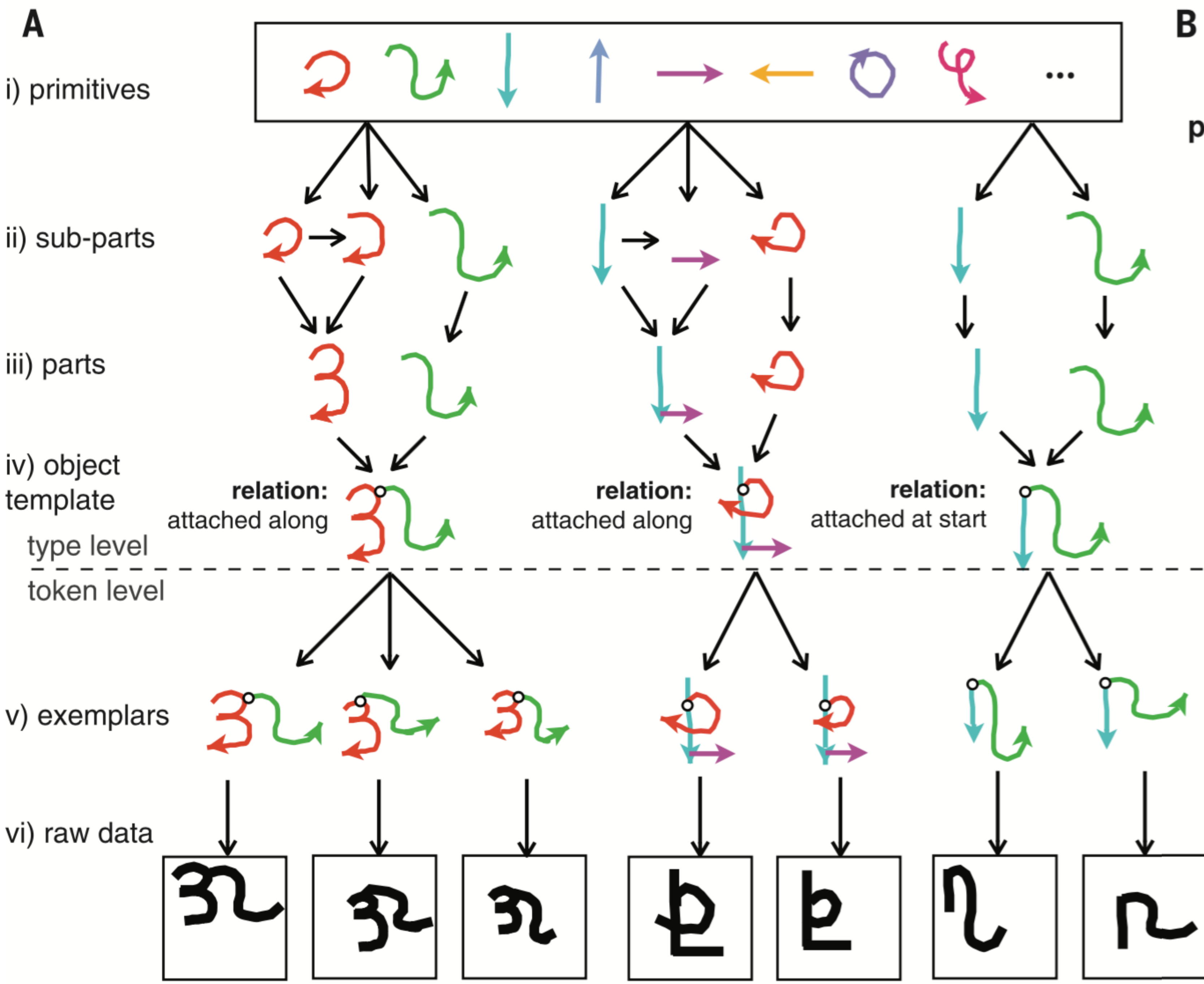


[Battaglia et al., 2013]

Can your phone improve your health? p. 1306
Vitamin C and cancer cell metabolism pp. 1317 & 1391
In search of stem cells p. 1319
Science
$10
11 DECEMBER 2015
sciencemag.org
AAAS
Machine intelligence
AI learns to read and write letters p. 1332



[Lake et al, 2016]



B

procedure GENERATETYPE
  $\kappa \leftarrow P(\kappa)$ ▷ Sample number of parts
  for i = 1 ... κ **do**
    $n_i \leftarrow P(n_i|\kappa)$ ▷ Sample number of sub-parts
    for j = 1 ... n_i **do**
      $s_{ij} \leftarrow P(s_{ij}|s_{i(j-1)})$ ▷ Sample sub-part sequence
    end for
    $R_i \leftarrow P(R_i|S_1, ..., S_{i-1})$ ▷ Sample relation
  end for
  $\psi \leftarrow \{\kappa, R, S\}$
  return @GENERATETOKEN(ψ) ▷ Return program

procedure GENERATETOKEN(ψ)
  for $i = 1...\kappa$ **do**
    $S_i^{(m)} \leftarrow P(S_i^{(m)}|S_i)$ ▷ Add motor variance
    $L_i^{(m)} \leftarrow P(L_i^{(m)}|R_i, T_1^{(m)}, ..., T_{i-1}^{(m)})$
      ▷ Sample part's start location
    $T_i^{(m)} \leftarrow f(L_i^{(m)}, S_i^{(m)})$ ▷ Compose a part's trajectory
  end for
  $A^{(m)} \leftarrow P(A^{(m)})$ ▷ Sample affine transform
  $I^{(m)} \leftarrow P(I^{(m)}|T^{(m)}, A^{(m)})$ ▷ Sample image
  return $I^{(m)}$

[Lake et al, 2016]

One-knower

```
λ S . (if (singleton? S)
          "one"
          undef)
```

Two-knower

```
λ S . (if (singleton? S)
          "one"
          (if (doubleton? S)
              "two"
              undef))
```

Three-knower

```
λ S . (if (singleton? S)
          "one"
          (if (doubleton? S)
              "two"
              (if (tripleton? S)
                  "three"
                  undef))
```

CP-knower

```
λ S . (if (singleton? S)
          "one"
          (next (L (set-difference S
                                   (select S)))))
```

Singular-Plural

```
λ S . (if (singleton? S)
          "one"
          "two")
```

Mod-5

```
λ S . (if (or (singleton? S)
              (equal-word? (L (set-difference S)
                                  (select S))
                           "five"))
          "one"
          (next (L (set-difference S
                                   (select S)))))
```

2-not-1-knower

```
λ S . (if (doubleton? S)
          "two"
          undef)
```

2N-knower

```
λ S . (if (singleton? S)
          "one"
          (next (next (L (set-difference S (select S))))))
```

[Piantadosi et al, 2012]

summary

- knowledge of environment can be represented in physical systems
- we can use formal systems for an abstract description of systems
- due to the universality of computation, we can use formal systems that are optimised for computational problems that the brain has to solve
- classical logic can only handle true/false statements, but most things we are interested in are uncertain
- logic can be extended to handle uncertain knowledge, by allowing truth values to be between true and false (degree of belief/plausibility)
    - it is advantageous if beliefs form a probability measure
    - for realistic numbers of variables this direct representation through probability tables is often not feasible
    - compositionality (infinite use of finite building blocks) can be used to mitigate this issue (e.g. graphical models)
- universal probabilistic languages

key concepts

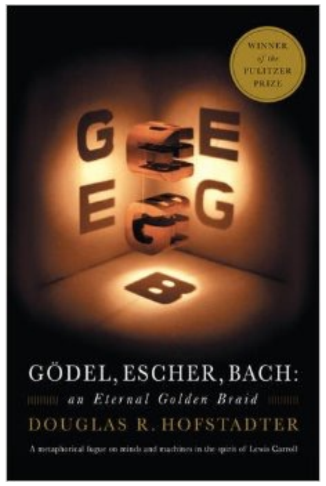
- computability, universal Turing-machine
- generative model (probabilistic environment simulator)
- probability, conditional probability
- probabilistic inference as conditional probability

supporting material

davidnagy.web.elte.hu/references/knowledgerep_thesisexcerpt.pdf

formal systems

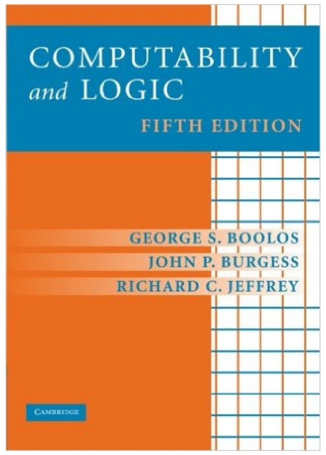
D. Hofstadter
Gödel Escher Bach



L.E. Szabó 2007
Bevezetés a matematikai logikába
http://philosophy.elte.hu/leszabo/Logika/logika.pdf

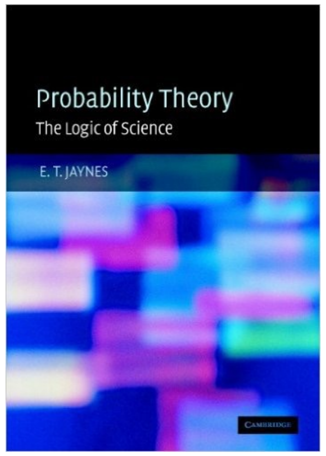
universality of computation

G. Boolos, J.P. Burgess, R.C. Jeffrey
Computability and Logic



logical interpretation of probability theory

E. T. Jaynes
Probability Theory



probabilistic programs

N. Goodman
Probabilistic Programs: A New Language for AI
https://www.youtube.com/watch?v=fcIvsoaUI-U

videos

Antikythera mechanism
https://www.youtube.com/watch?v=UpLcnAIpVRA

Lego antikythera
https://www.youtube.com/watch?v=RLPVCJjTNgk

Babbage's difference engine
https://www.youtube.com/watch?v=jiRgdaknJCg

Universal Turing machine from lego
https://www.youtube.com/watch?v=KrNTmOSVW-U

Universal Turing machine in game of life
https://www.youtube.com/watch?v=My8AsV7bA94

assignments

- running the Turing machine given as an example on an empty tape (filled with 0s), starting from state A, what happens in the first 5 steps? (what are the tape states?)

Tape symbol	Current state A			Current state B			Current state C		
	Write symbol	Move tape	Next state	Write symbol	Move tape	Next state	Write symbol	Move tape	Next state
0	1	R	**B**	1	L	**A**	1	L	**B**
1	1	L	**C**	1	R	**B**	1	R	**HALT**

- in the introduction of graphic models, we said that, in the general case, $2^n - 1$ numbers are required to specify the entire distribution, where n is the number of variables. Why?
- in the graphic model given as an example (slide 86), is it possible that subject has the flu, but doesn't sneeze?
- in the example logical system (slide 67) can it happen that subject is sick but does not cough?